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Physics School 2010:
B Physics

Lecture 1: UT angles I (Belle/BaBar)
Lecture 2: UT angles II (Belle/BaBar)
Lecture 3: UT sides (Belle/BaBar/CDF/D0)
Lecture 4: Future Super-b Factories

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University of Bern

Some caveats:

- *will not talk much about history*
- *will not talk much about experimental technical details*
- *will not cover all experiments equally (Belle bias)*
- *may not cover your favorite topic*
- *will probably run out of time*

Why study flavor physics?

Big ***issues:***

- ***why $SU(2)_L xU(1)$?***
- ***what breaks $SU(2)_L xU(1)$?***
- ***what gives particle mass?***
- ***what stabilizes the electroweak scale below 1 TeV?***

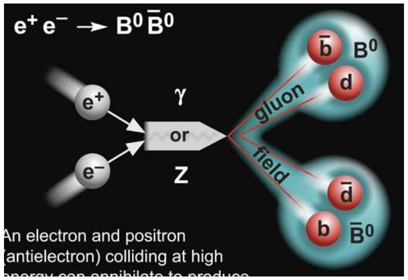
⇒ ***LHC***
(Atlas, CMS)
(i.e., the "energy frontier")

but let's not forget:

- ***why 3 generations? (are there more?)***
- ***why are the masses so different?***
- ***why the pattern of CKM weak couplings?***
- ***what causes the phase in the CKM matrix?***
- ***why do we live in a matter, rather than antimatter, universe?***

⇒ ***Flavor "factory":***
(CLEO, Belle, BaBar, CDF/D0, BESIII, Belle-II, SuperB, LHCb)
(i.e., a facility where large numbers of heavy quarks (c,b) or leptons (τ) are produced)

Reminder:
solutions to the latter set may help us answer the first set, and vice-versa



The Standard Model

Fermions:

FERMIONS matter constituents spin = 1/2, 3/2, 5/2, …

Leptons spin = 1/2

Flavor	Mass GeV/c^2	Electric charge
ν_L lightest neutrino*	$(0-0.13)\times10^{-9}$	0
e electron	0.000511	–1
ν_M middle neutrino*	$(0.009-0.13)\times10^{-9}$	0
μ muon	0.106	–1
ν_H heaviest neutrino*	$(0.04-0.14)\times10^{-9}$	0
τ tau	1.777	–1

Quarks spin = 1/2

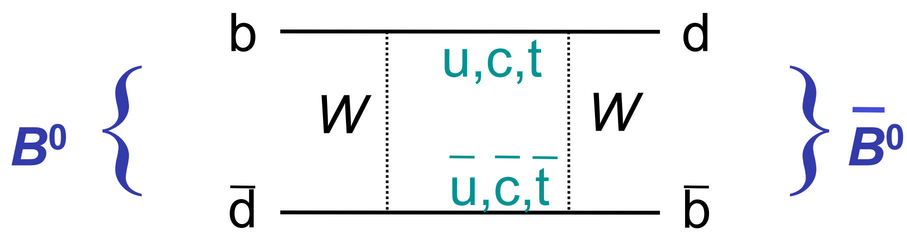
Flavor	Approx. Mass GeV/c^2	Electric charge	
u up	0.002	2/3	1st doublet
d down	0.005	–1/3	
c charm	1.3	2/3	2nd doublet
s strange	0.1	–1/3	
t top	173	2/3	3rd doublet
b bottom	4.2	–1/3	

Mesons:

Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
π^+	pion	$u\bar{d}$	+1	0.140	0
K^-	kaon	$s\bar{u}$	–1	0.494	0
ρ^+	rho	$u\bar{d}$	+1	0.776	1
B^0	B-zero	$d\bar{b}$	0	5.279	0
η_c	eta-c	$c\bar{c}$	0	2.980	0

Quantum mechanics: neutral meson mixing

The Standard Model of particle interactions prescribe interactions such as



Since B^0 and $\overline{B}^0$ are mass-degenerate, they form a 2-D basis of the Hamiltonian, and the eigenstates of the Hamiltonian (mass eigenstates) are in general comprised of both flavor eigenstates.

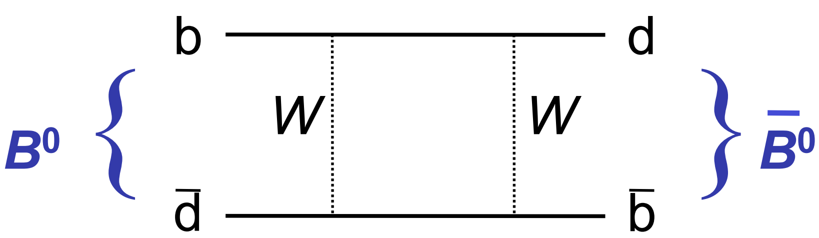
⇒ Mass eigenstates are not flavor eigenstates.

$$i\frac{\partial}{\partial t}\begin{pmatrix} |B^0\rangle \\ |\overline{B}^0\rangle \end{pmatrix} = \left(\mathrm{M} - \frac{i}{2}\Gamma\right)\begin{pmatrix} |B^0\rangle \\ |\overline{B}^0\rangle \end{pmatrix}$$

$$\begin{aligned} |B_H\rangle &= p|B^0\rangle - q|\overline{B}^0\rangle \\ |B_L\rangle &= p|B^0\rangle + q|\overline{B}^0\rangle \end{aligned}$$

$$\begin{aligned} |B_H(t)\rangle &= e^{(im_H - \Gamma_H/2)t}\,|B_H\rangle \\ |B_L(t)\rangle &= e^{(im_L - \Gamma_L/2)t}\,|B_L\rangle \end{aligned}$$

Neutral meson mixing, cont'd



$$|B_H\rangle = p|B^0\rangle - q|\overline{B}^0\rangle$$
$$|B_L\rangle = p|B^0\rangle + q|\overline{B}^0\rangle$$

$$|B_H(t)\rangle = e^{(im_H - \Gamma_H/2)t}|B_H\rangle$$
$$|B_L(t)\rangle = e^{(im_L - \Gamma_L/2)t}|B_L\rangle$$

$$|B^0\rangle = \frac{1}{2p}\Big(|B_L\rangle + |B_H\rangle\Big)$$
$$|\overline{B}^0\rangle = \frac{1}{2q}\Big(|B_L\rangle - |B_H\rangle\Big)$$

$$|B^0(t)\rangle = \frac{1}{2p}\left\{|B_L\rangle e^{-(\Gamma_L/2+im_L)t} + |B_H\rangle e^{-(\Gamma_H/2+im_H)t}\right\}$$
$$= e^{-(\overline{\Gamma}/2+i\overline{m})\,t}\left\{\cosh\left[(\Delta\gamma/4 + i\Delta m/2)t\right]|B^0\rangle + \left(\frac{q}{p}\right)\sinh\left[(\Delta\gamma/4 + i\Delta m/2)t\right]|\overline{B}^0\rangle\right\}$$

Possible CP violation

oscillations/mixing

$$|\overline{B}^0(t)\rangle = e^{-(\overline{\Gamma}/2+i\overline{m})\,t}\left\{\left(\frac{p}{q}\right)\sinh\left[(\Delta\gamma/4 + i\Delta m/2)t\right]|B^0\rangle + \cosh\left[(\Delta\gamma/4 + i\Delta m/2)t\right]|\overline{B}^0\rangle\right\}$$

$$\overline{\Gamma} \equiv \frac{1}{2}(\Gamma_H + \Gamma_L) \qquad \overline{m} \equiv \frac{1}{2}(m_H + m_L) \qquad \Delta\gamma \equiv \Gamma_H - \Gamma_L \qquad \Delta m \equiv m_H - m_L$$

Neutral meson mixing, cont'd

Neglect CP violation (q/p=1), consider a final state reachable only via B^0 or $\overline{B}^0$, i.e., semileptonic decay: $B^0 \to D^-\ell^+\nu$ and $\overline{B}^0 \to D^+\ell^-\nu$. No CPV implies $\mathcal{A}(B^0 \to D^-\ell^+\nu) = \mathcal{A}(\overline{B}^0 \to D^+\ell^-\nu)$ and thus:

$$
\begin{aligned}
|\langle D^-\ell^+\nu|H|B^0(t)\rangle|^2 &= \frac{|\mathcal{A}|^2 e^{-\Gamma t}}{2}\,|\cosh(\Delta\gamma/4 + i\Delta m/2)t\,|^2 \\
&= \frac{|\mathcal{A}|^2 e^{-\Gamma t}}{4}\,[\cosh(\Delta\gamma/2)t + \cos(\Delta m)t\,] \\
&\approx \frac{|\mathcal{A}|^2 e^{-\Gamma t}}{4}\,[1 + \cos(\Delta m)t\,] \quad \text{in } B^0\text{-}\overline{B}^0 \text{ system}
\end{aligned}
$$

$$
\begin{aligned}
|\langle D^+\ell^-\nu|H|B^0(t)\rangle|^2 &= \frac{|\mathcal{A}|^2 e^{-\Gamma t}}{2}\,|\sinh(\Delta\gamma/4 + i\Delta m/2)t\,|^2 \\
&= \frac{|\mathcal{A}|^2 e^{-\Gamma t}}{4}\,[\cosh(\Delta\gamma/2)t - \cos(\Delta m)t\,] \\
&\approx \frac{|\mathcal{A}|^2 e^{-\Gamma t}}{4}\,[1 - \cos(\Delta m)t\,] \quad \text{in } B^0\text{-}\overline{B}^0 \text{ system}
\end{aligned}
$$

⇒ fitting the decay time distributions of "right-sign" and "wrong-sign" decays allows us to determine the parameter Γ (decay width=1/τ) and Δm

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Neutral meson mixing, cont'd

In fact, one does not need to measure the decay time dependence; one can simply count events:

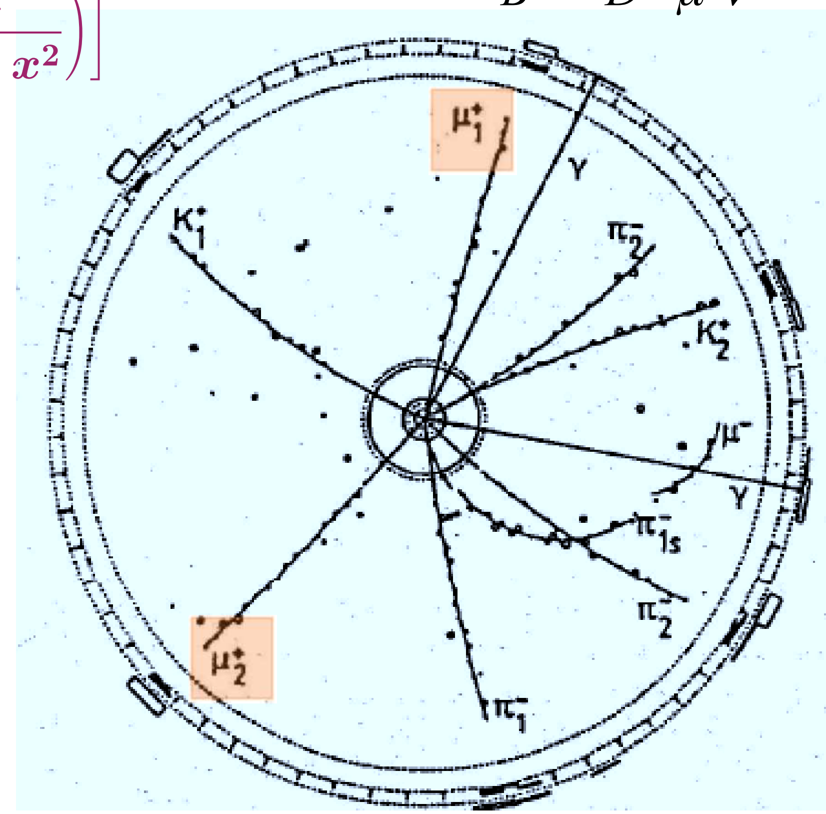
$$\int_0^\infty |\langle D^-\ell^+\nu|H|B^0(t)\rangle|^2\,dt = \frac{|\mathcal{A}|^2 e^{-\Gamma t}}{4}\left[\frac{1}{\Gamma}+\frac{1}{\Gamma}\left(\frac{1}{1+x^2}\right)\right] = \frac{|\mathcal{A}|^2 e^{-\Gamma t}}{4\Gamma}\left(\frac{2+x^2}{1+x^2}\right)$$

$$\int_0^\infty |\langle D^+\ell^-\nu|H|B^0(t)\rangle|^2\,dt = \frac{|\mathcal{A}|^2 e^{-\Gamma t}}{4}\left[\frac{1}{\Gamma}-\frac{1}{\Gamma}\left(\frac{1}{1+x^2}\right)\right] = \frac{|\mathcal{A}|^2 e^{-\Gamma t}}{4\Gamma}\left(\frac{x^2}{1+x^2}\right)$$

$$\Rightarrow \quad \chi \equiv \frac{N^{\text{wrong sign}}}{N^{\text{total}}} = \frac{x^2}{2(1+x^2)}$$

$\bar{B}^0 \rightarrow D^{*-}\mu^+\nu$

$B^0 \rightarrow D^{*-}\mu^+\nu$



This is how B^0-B^0 mixing was discovered, a time-independent measurement by ARGUS at DESY *[Albrecht et al., PLB 192, 245 (1987)]*:

χ_d = 0.17 ± 0.05
(25 like-sign events from 0.10 fb^{-1})

Neutral meson mixing, cont'd

Hastings et al. (Belle), PRD 67, 052004 (2003) ***[29 fb^{-1}]***

Select dilepton ($\mu\mu$, ee) events:

$M_{e+\,e-} > 100$ MeV/c^2,
1.1 GeV/c $< p^* < 2.3$ GeV/c

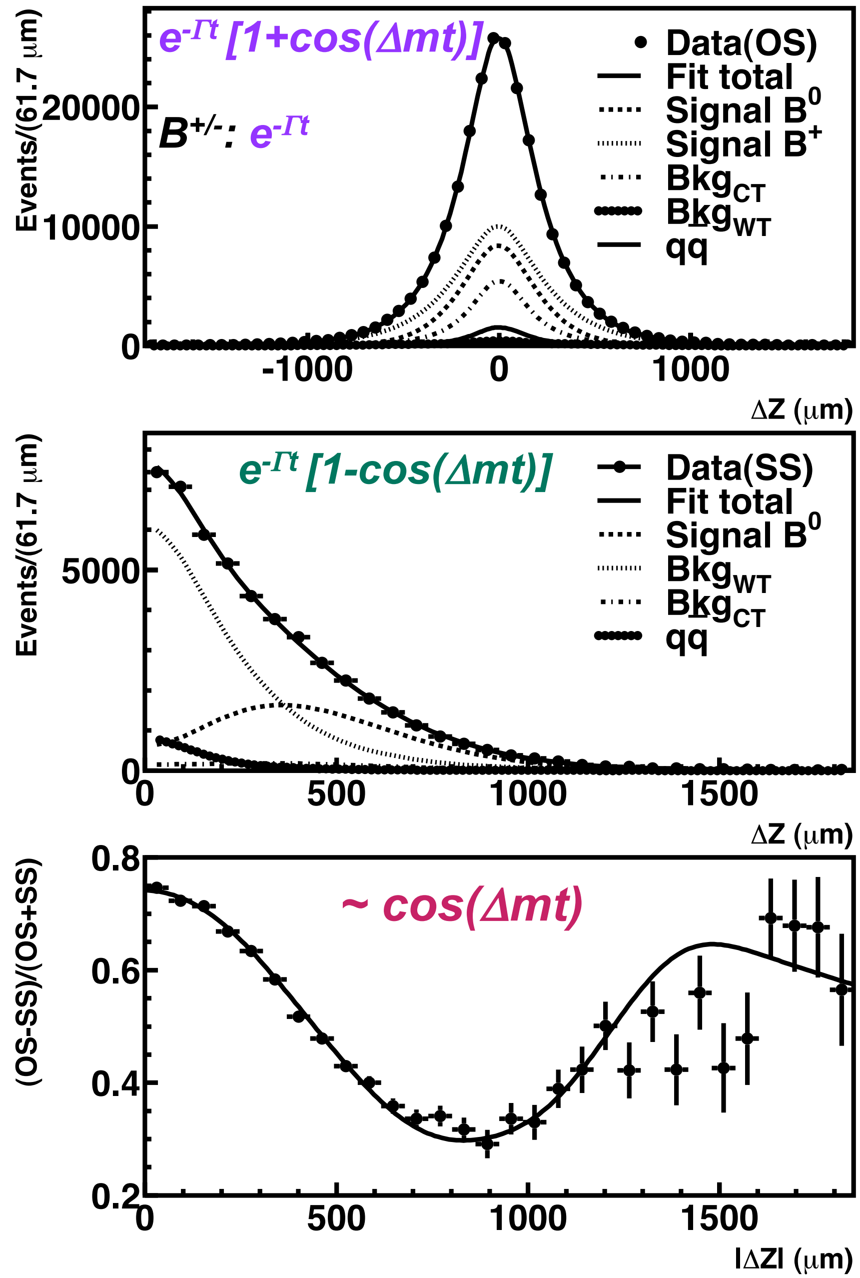
$-0.15 < (M_{e+\,e-} - M_{J/\psi}) < 0.05$ GeV/c^2
$|M_{\mu+\mu-} - M_{J/\psi}) < 0.05$ GeV/c^2

$\Rightarrow$
49838 same-sign (SS) events
230881 opposite-sign (OS) events

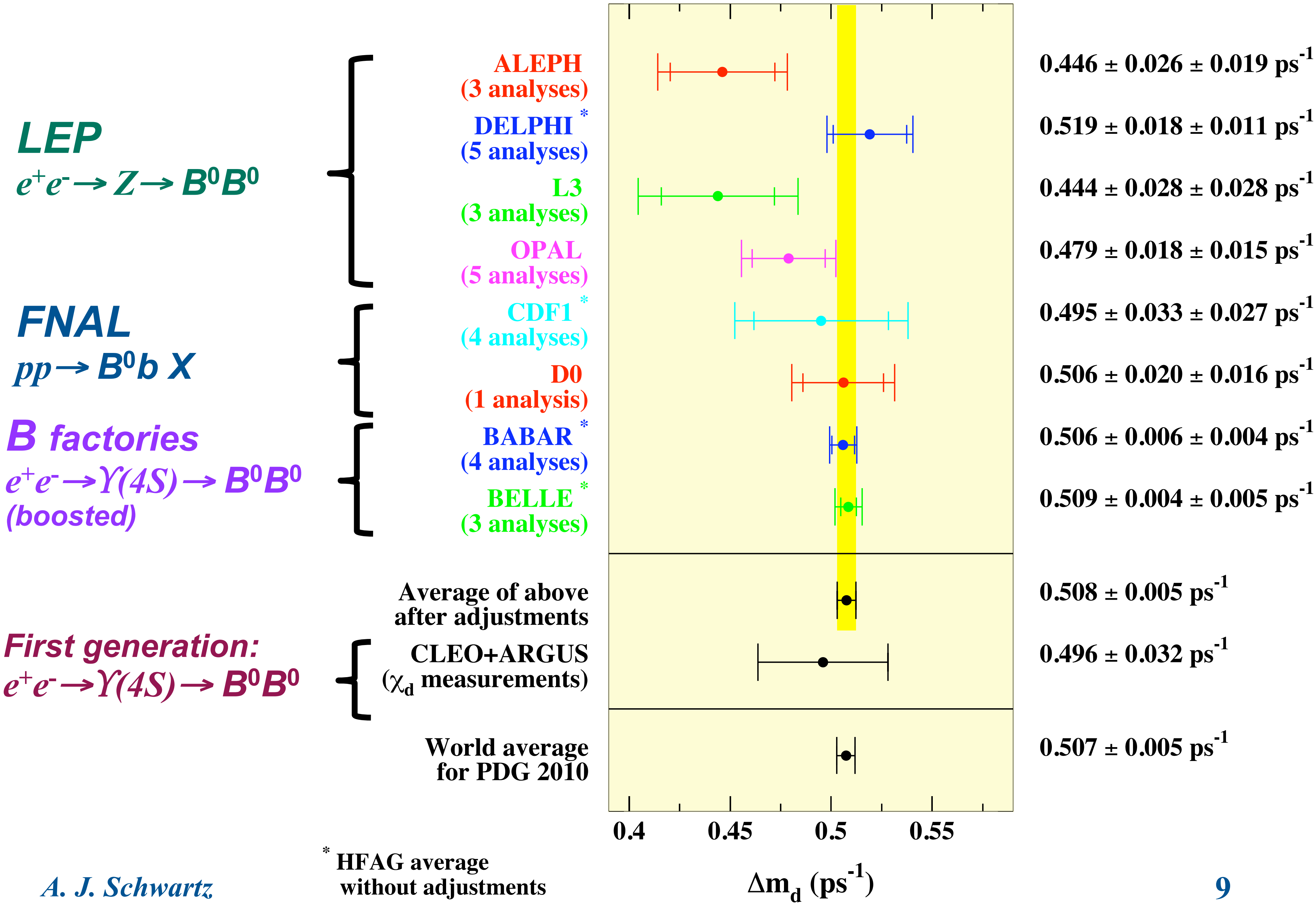
Simultaneously fit samples, taking $\tau = 1.542 \pm 0.016$ ps (PDG):

$\Delta m_d = (0.503 \pm 0.008 \pm 0.010)\ \text{ps}^{-1}$

$f_+/f_0 = 1.01 \pm 0.03 \pm 0.09$

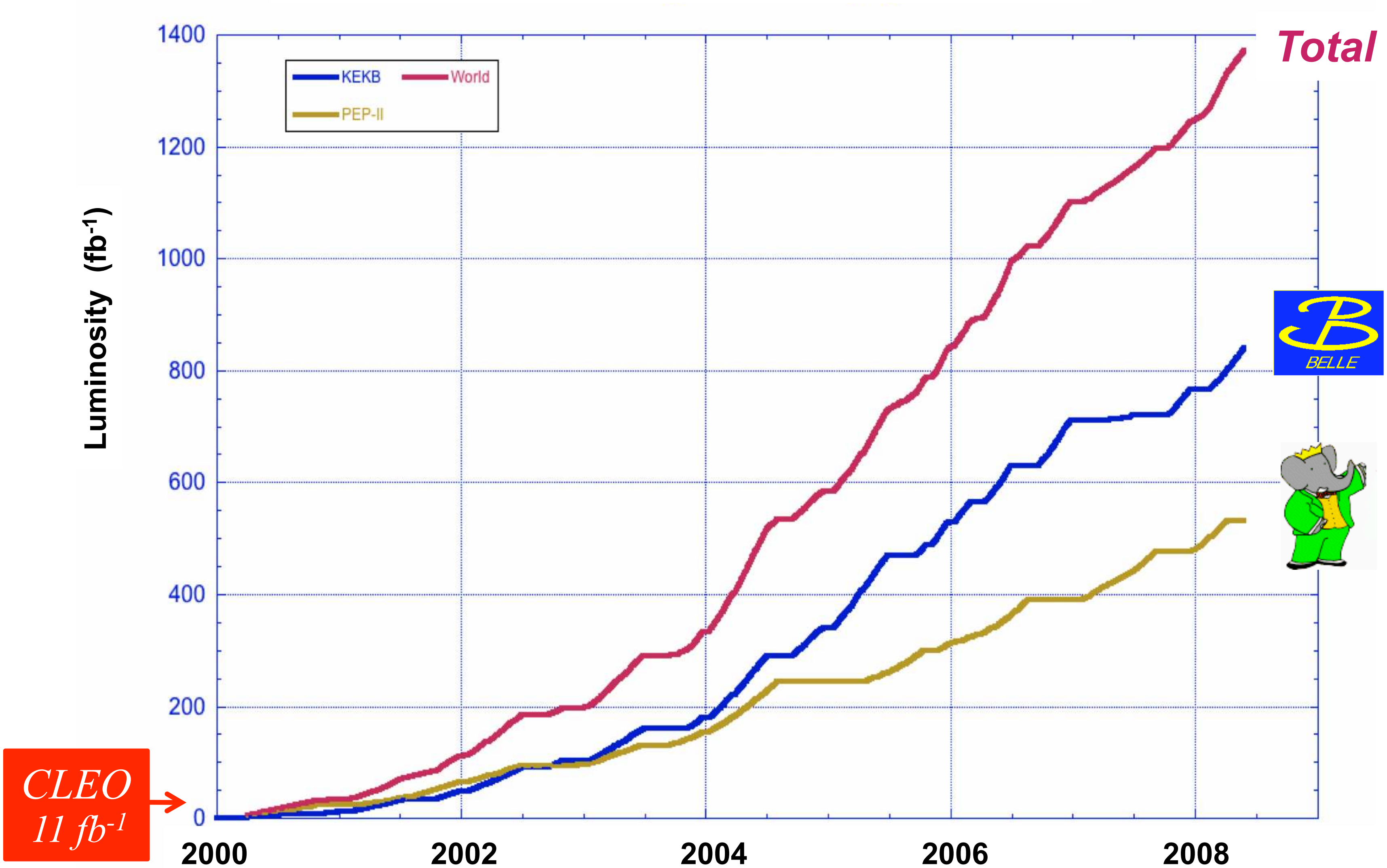


Heavy Flavor Averaging Group (HFAG) World Average



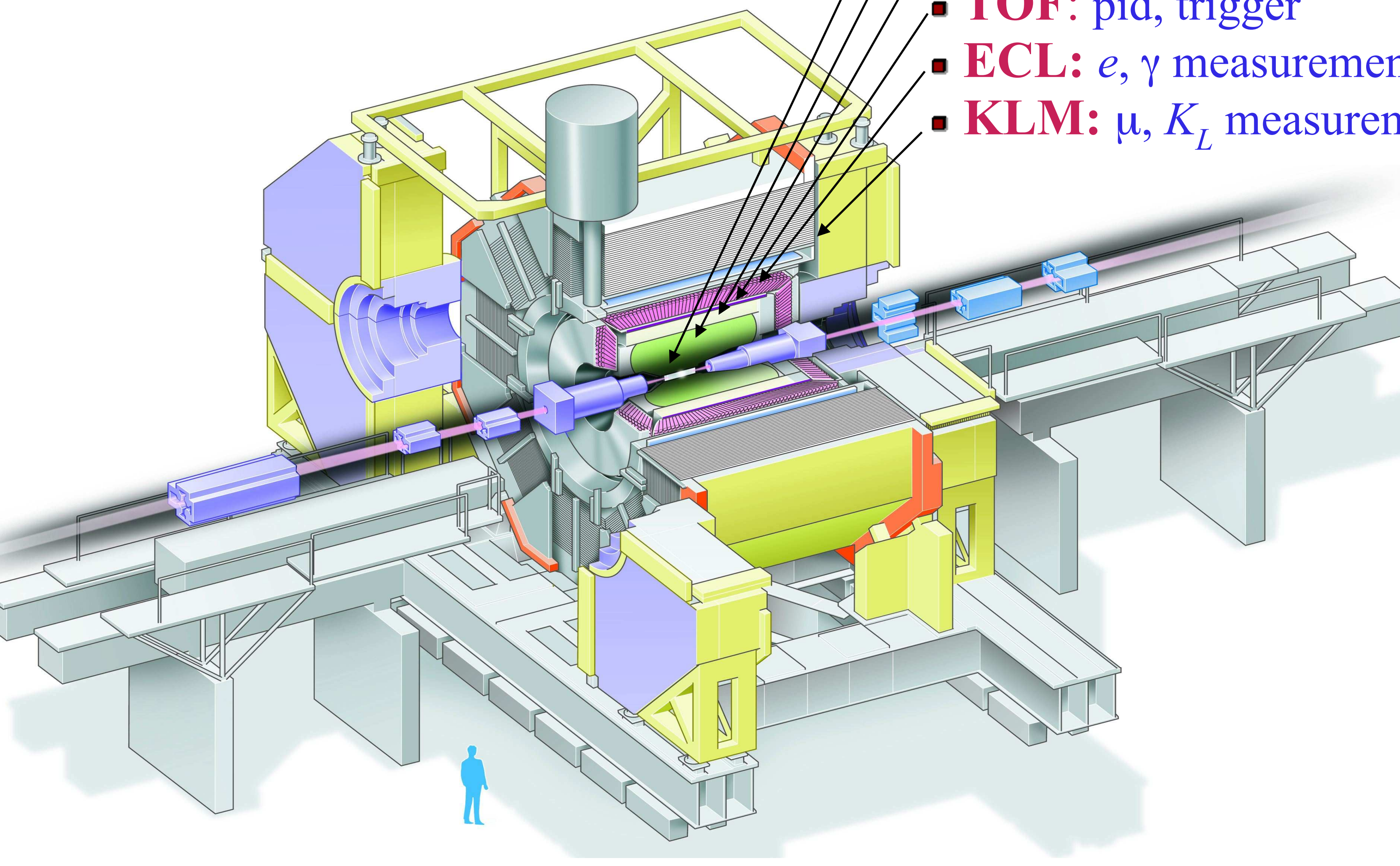
B factory performance to-date:

World integrated luminosity on $\Upsilon(4S)$:



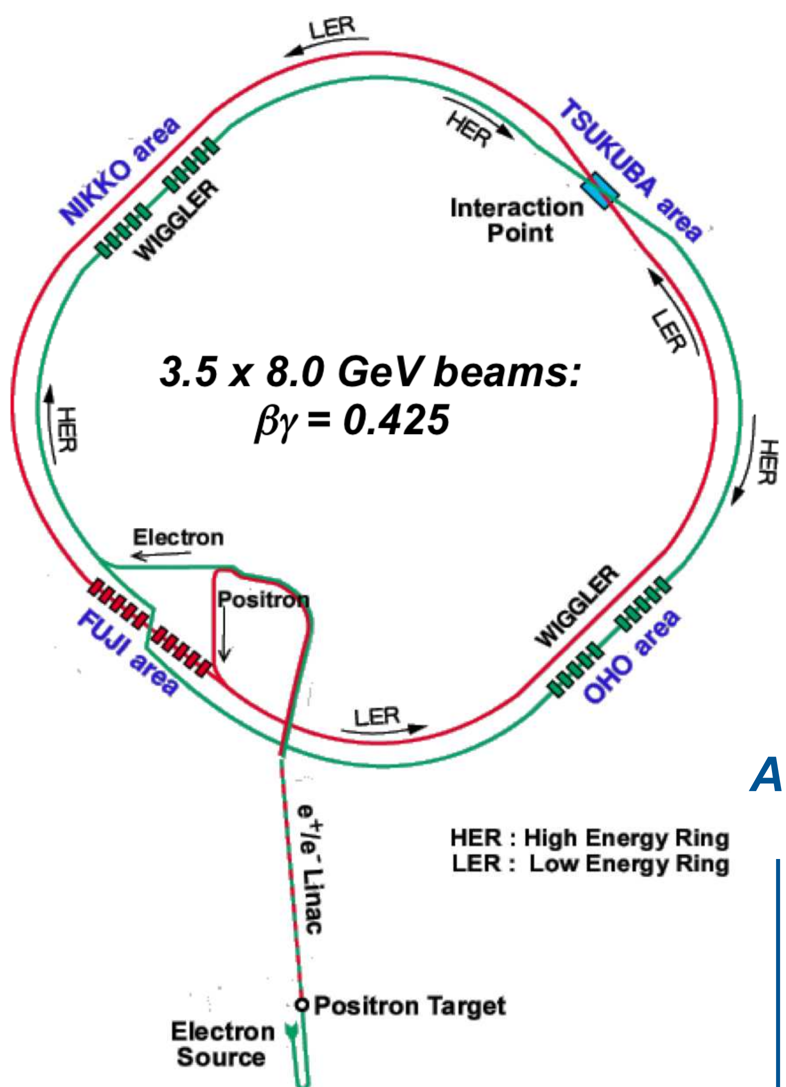
The Belle Detector:

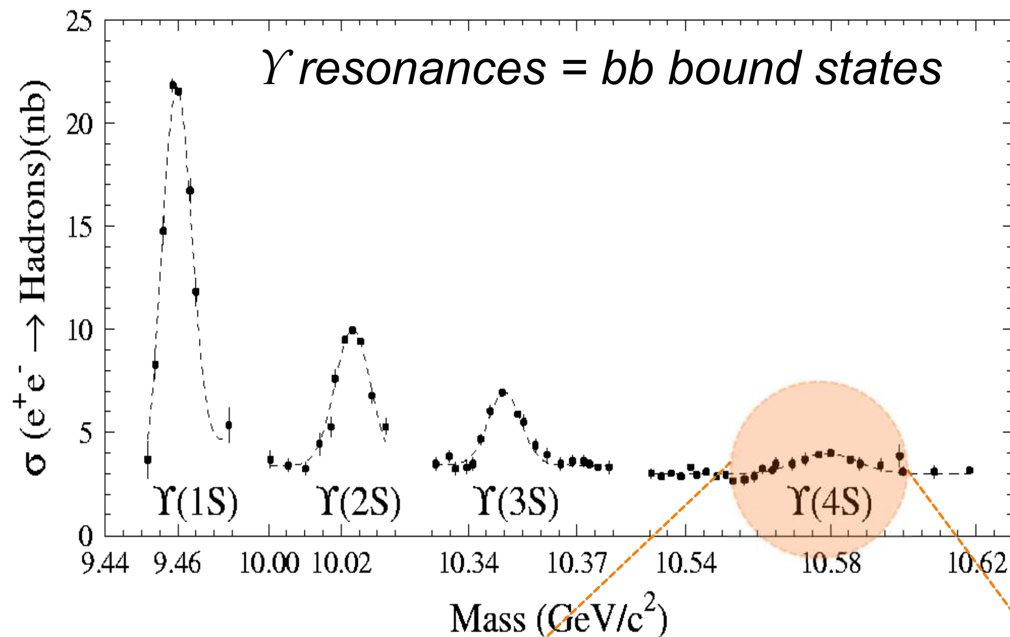
- **SVD:** vertexing (lifetime)
- **CDC:** tracking, *dE/dx* for pid
- **ACC:** aerogel Cerenk. Counter
- **TOF**: pid, trigger
- **ECL:** *e*, γ measurement
- **KLM:** μ, K_L measurement

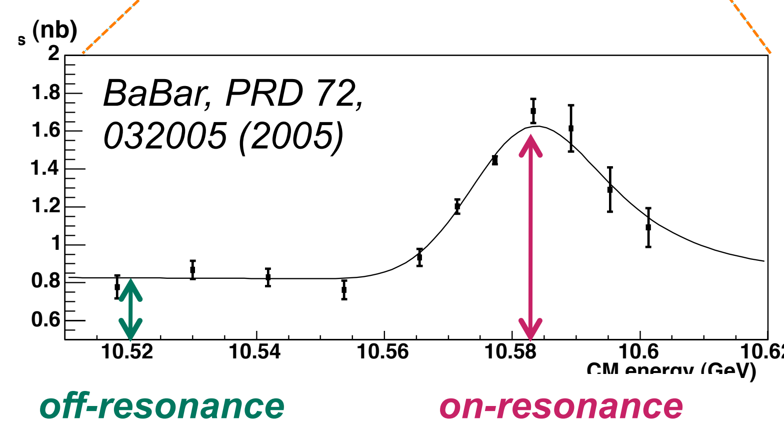


KEKB: running at the $\Upsilon(4S)$ resonance

KEKB collider:







At $\Upsilon(4S)$ resonance:
($\sqrt{s}$ = 10.579 GeV)

$\sigma(bb) \approx 1.1$ nb
$\sigma(cc) \approx 1.3$ nb
$\sigma(uu) \approx 1.4$ nb
$\sigma(dd,ss) \approx 0.3$ nb

Typical Analysis Steps:

1) $B \rightarrow f$ selection:

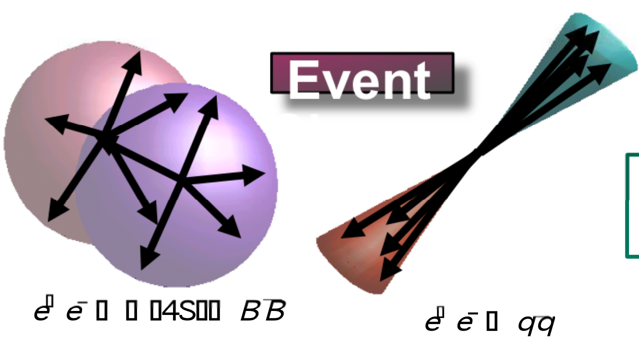
$$M_{bc} \equiv \sqrt{E^2_{\rm beam} - p^2_B}$$
$$\Delta E \equiv E_B - E_{\rm beam}$$

(e.g., for $B \rightarrow \pi^+\pi^-$:
5.271 < m_{bc} < 5.287 GeV/c^2
$|\Delta E|$ < 0.064 GeV)

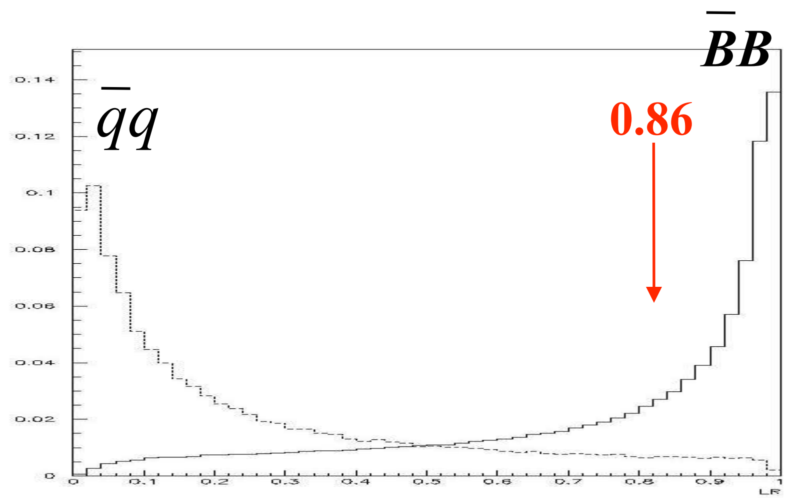
2) Flavor tagging:

mainly $K^{\pm}, \mu^{\pm}, e^{\pm}$ output: $q = \pm 1$, quality r = 0-1

3) Continuum suppression:



$KLR = \mathcal{L}_{BB}/(\mathcal{L}_{BB}+\mathcal{L}_{qq})$

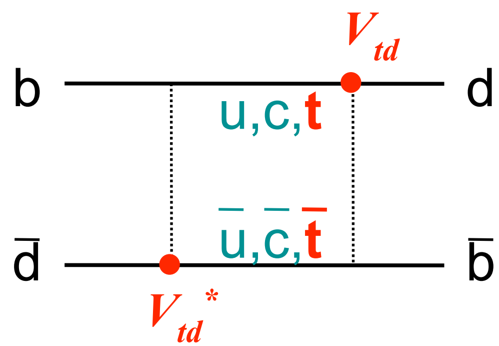


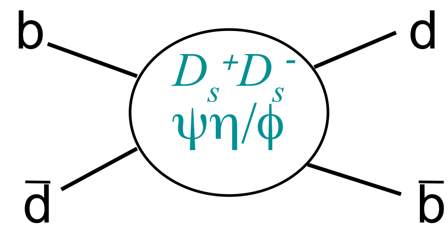
KLR

4) Vertexing and Δt fit: $\Delta z_{lab} = \gamma\beta c\, \Delta t_B$

Neutral meson mixing, cont'd

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$$





off-shell ("virtual") states: Δm *on-shell states:* $\Delta\Gamma$

Meson	flavors	$\Delta m/\Gamma$	$\Delta\Gamma/2\Gamma$	when mixing observed
K^0	$\bar{s}d$	0.474	0.997	1958
B^0	$\bar{b}d$	0.773	< 1%	1987
B_s^0	$\bar{b}s$	27	0.15 ±0.07	2006
D^0	$\bar{c}u$	< 0.029	0.011±0.005	2007

B^0-$\bar{B}^0$ system, allowing for CP violation:

$$|B_H\rangle = p\,|B^0\rangle - q\,|\overline{B}^0\rangle$$
$$|B_L\rangle = p\,|B^0\rangle + q\,|\overline{B}^0\rangle$$

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - (i/2)\Gamma_{12}^*}{M_{12} - (i/2)\Gamma_{12}}} \approx \sqrt{\frac{M_{12}^*}{M_{12}}} = e^{i2\phi_1} \qquad \text{(phase of } V_{td}^* V_{tb}\text{)}$$

$$|B^0(t)\rangle = e^{-(\Gamma/2+i\bar{m})\,t}\left[\cos\left(\frac{\Delta m}{2}t\right)|B^0\rangle + \left(\frac{q}{p}\right) i\sin\left(\frac{\Delta m}{2}t\right)|\bar{B}^0\rangle\right]$$
$$|\bar{B}^0(t)\rangle = e^{-(\Gamma/2+i\bar{m})\,t}\left[\left(\frac{p}{q}\right) i\sin\left(\frac{\Delta m}{2}t\right)|B^0\rangle + \cos\left(\frac{\Delta m}{2}t\right)|\bar{B}^0\rangle\right],$$

$$|\langle f|H|B^0(t)\rangle|^2 = \frac{|\mathcal{A}_f|^2 e^{-\Gamma t}}{2}\left[1 + |\lambda|^2 + (1-|\lambda|^2)\cos(\Delta m t) - 2\,\mathrm{Im}\,\lambda\,\sin(\Delta m t)\right]$$

$$|\langle \bar{f}|H|\bar{B}^0(t)\rangle|^2 = \frac{|\bar{\mathcal{A}}_{\bar{f}}|^2 e^{-\Gamma t}}{2}\left[1 + |\bar{\lambda}|^2 + (1-|\bar{\lambda}|^2)\cos(\Delta m t) - 2\,\mathrm{Im}\,\bar{\lambda}\,\sin(\Delta m t)\right]$$

$$\lambda = \left(\frac{q}{p}\right)\frac{A(\overline{B}^0 \to f)}{A(B^0 \to f)} \qquad \bar{\lambda} = \left(\frac{p}{q}\right)\frac{A(B^0 \to \bar{f})}{A(\overline{B}^0 \to \bar{f})}$$

B^0-$\bar{B}^0$ system, allowing for CP violation:

$$|\langle f|H|B^0(t)\rangle|^2 = \frac{|\mathcal{A}_f|^2 e^{-\Gamma t}}{2}\left[1+|\lambda|^2+(1-|\lambda|^2)\cos(\Delta m t)-2\,\mathrm{Im}\,\lambda\,\sin(\Delta m t)\right]$$

$$|\langle \bar{f}|H|\bar{B}^0(t)\rangle|^2 = \frac{|\bar{\mathcal{A}}_{\bar{f}}|^2 e^{-\Gamma t}}{2}\left[1+|\bar{\lambda}|^2+(1-|\bar{\lambda}|^2)\cos(\Delta m t)-2\,\mathrm{Im}\,\bar{\lambda}\,\sin(\Delta m t)\right]$$

$$\lambda = \left(\frac{q}{p}\right)\frac{A(\overline{B}^0\to f)}{A(B^0\to f)} \qquad \bar{\lambda} = \left(\frac{p}{q}\right)\frac{A(B^0\to \bar{f})}{A(\overline{B}^0\to \bar{f})}$$

3 types of CPV:

$\lvert A(B^0\to f)\rvert \neq \lvert A(\overline{B}^0\to \bar{f})\rvert$ or $\lvert A(B^0\to \bar{f})\rvert \neq \lvert A(\overline{B}^0\to f)\rvert$	direct CPV
$\left\lvert\frac{q}{p}\right\rvert \neq 1$	CPV in mixing
$\mathrm{Im}\lambda \neq 0$	CPV in interference between direct and mixed amplitude

B^0-$\bar{B}^0$ system, CP violation:

In B^0-$\bar{B}^0$ system, $|q/p| \approx 1$. Now choose CP (self-conjugate) final state:

$$|\langle f|H|B^0(t)\rangle|^2 = \frac{|\mathcal{A}_f|^2 e^{-\Gamma t}}{2}\left[1+|\lambda|^2+(1-|\lambda|^2)\cos(\Delta m t) - 2\,\mathrm{Im}\,\lambda\,\sin(\Delta m t)\right]$$

$$|\langle f|H|\bar{B}^0(t)\rangle|^2 = \frac{|\mathcal{A}_f|^2 e^{-\Gamma t}}{2}\left[1+|\lambda|^2-(1-|\lambda|^2)\cos(\Delta m t) + 2\,\mathrm{Im}\lambda\,\sin(\Delta m t)\right]$$

$$\frac{N_{\overline{B}^0\to f} - N_{B^0\to f}}{N_{\overline{B}^0\to f} + N_{B^0\to f}} = \mathcal{A}_f \cos(\Delta m\,\Delta t) + \mathcal{S}_f \sin(\Delta m\,\Delta t)$$

$$\mathcal{A}_f = \frac{1-|\lambda|^2}{1+|\lambda|^2} \qquad \mathcal{S}_f = \frac{2Im\,\lambda}{1+|\lambda|^2}$$

$$\lambda = \left(\frac{q}{p}\right)\frac{A(\overline{B}^0\to f)}{A(B^0\to f)} = e^{i2\phi_1}\,e^{i2\phi} \qquad \text{(one weak phase)}$$

$$\Rightarrow\ \mathcal{A}_f \approx 0, \qquad \mathcal{S}_f \approx \sin 2(\phi_1+\phi) = -\sin 2\phi'$$

The CKM matrix and Unitarity Triangle

All quark-quark coupling constants can be arranged in a matrix:

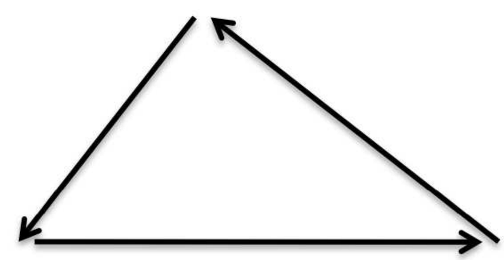
$$U \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

(columns: *d*, *s*, *b*; rows: *u*, *c*, *t*)

Unitarity ($U^\dagger U=1$) prescribes 6 complex equations:

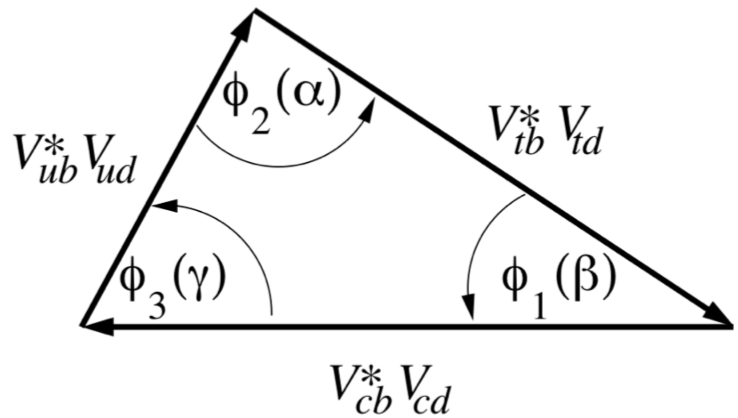
$$V_{ud}^* V_{cd} + V_{us}^* V_{cs} + V_{ub}^* V_{cb} = 0$$
$$V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} = 0$$
$$V_{cd}^* V_{td} + V_{cs}^* V_{ts} + V_{cb}^* V_{tb} = 0$$
$$V_{us}^* V_{ud} + V_{cs}^* V_{cd} + V_{ts}^* V_{td} = 0$$
$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$
$$V_{ub}^* V_{us} + V_{cb}^* V_{cs} + V_{tb}^* V_{ts} = 0$$

Each equation can be plotted in the complex plane as the sum of three vectors:



The Unitarity Triangle, cont'd

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$



The internal angles of this triangle are phase differences, which can be measured:

$$\phi_1\ (\beta) = \arg\left(\frac{V_{cb}^* V_{cd}}{-V_{tb}^* V_{td}}\right)$$

$$\phi_2\ (\alpha) = \arg\left(\frac{V_{tb}^* V_{td}}{-V_{ub}^* V_{ud}}\right)$$

$$\phi_3\ (\gamma) = \arg\left(\frac{V_{ub}^* V_{ud}}{-V_{cb}^* V_{cd}}\right)$$

Convention: V_{td} and V_{ub} are taken to be complex, others real

Measurement of ϕ_1 (β) with $B^0 \rightarrow J/\psi\, K^0$:

$$\lambda = \sqrt{\frac{M_{12}^*}{M_{12}}\,\frac{\bar{\mathcal{A}}_f}{\mathcal{A}_f}} = -\left(\frac{V_{td}V_{tb}^*}{V_{td}^*V_{tb}}\right)\left(\frac{V_{cb}\,V_{cs}^*}{V_{cb}^*\,V_{cs}}\right)\left(\frac{V_{cd}^*\,V_{cs}}{V_{cd}\,V_{cs}^*}\right)$$

$$= -\frac{V_{td}V_{tb}^*\,V_{cb}V_{cd}^*}{V_{td}^*V_{tb}\,V_{cb}^*V_{cd}}$$

$$= -\frac{-V_{cb}V_{cd}^*/(V_{td}^*V_{tb})}{-V_{cb}^*V_{cd}/(V_{td}V_{tb}^*)}$$

$$= -\frac{|\mathcal{M}|e^{-i\phi_1}}{|\mathcal{M}|e^{i\phi_1}}$$

$$= -e^{-2i\phi_1}$$

$$\Rightarrow \quad \mathcal{A}_{(J/\psi\,K^0)} = 0 \qquad \mathcal{S}_{(J/\psi\,K^0)} = \sin(2\phi_1)$$

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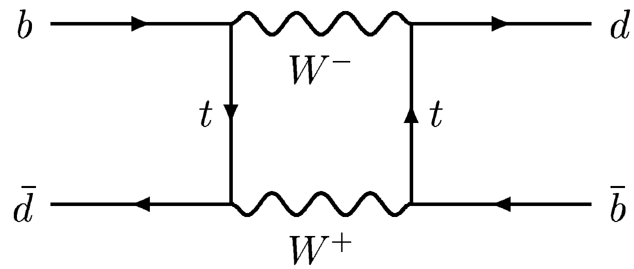
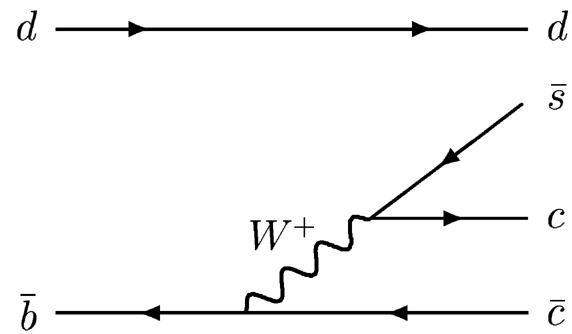
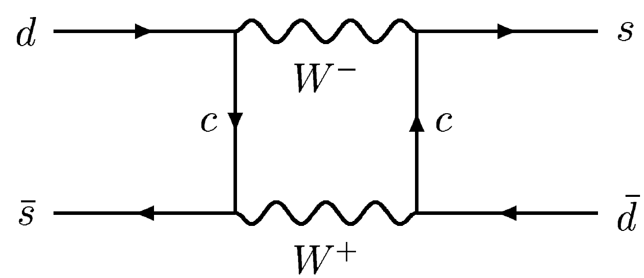
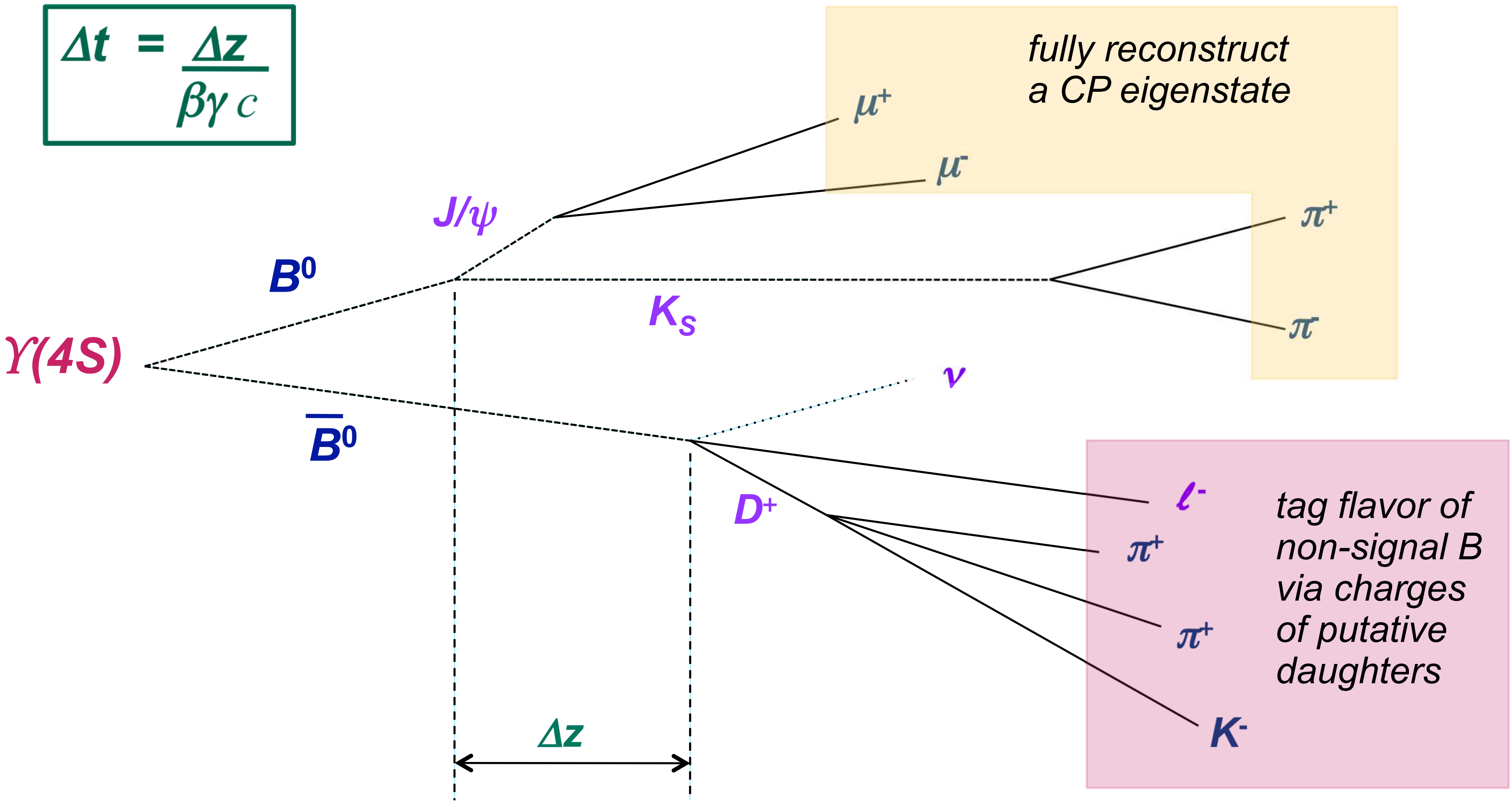






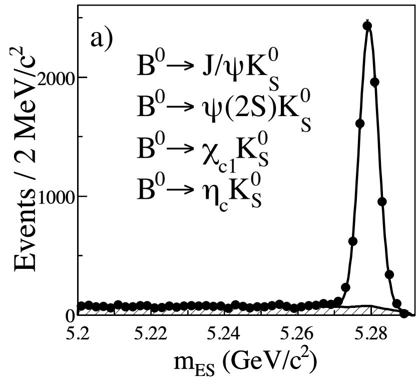
Decay time measurement:

$$\Delta t = \frac{\Delta z}{\beta\gamma\, c}$$

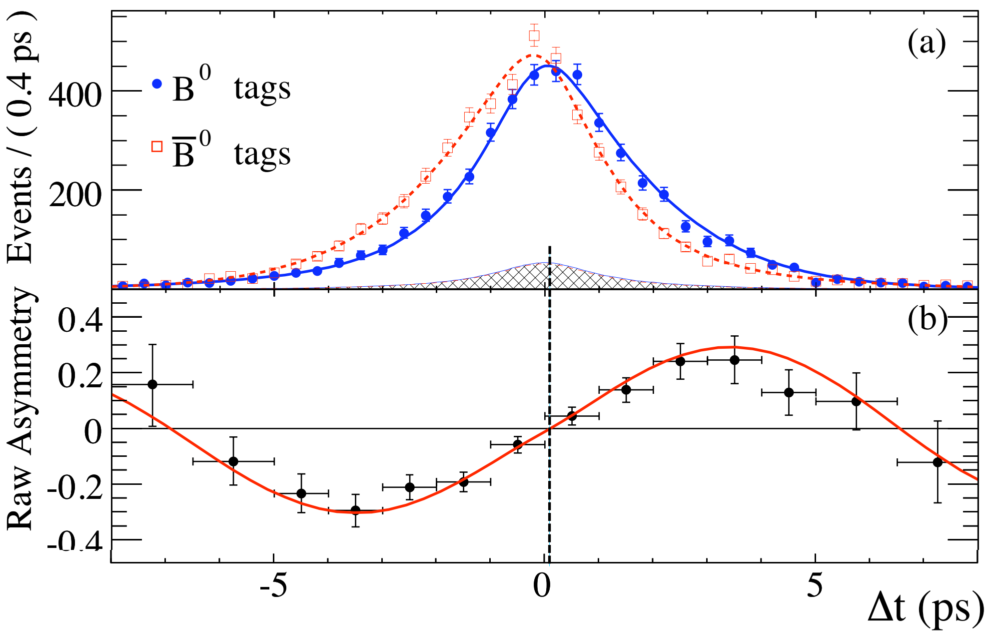


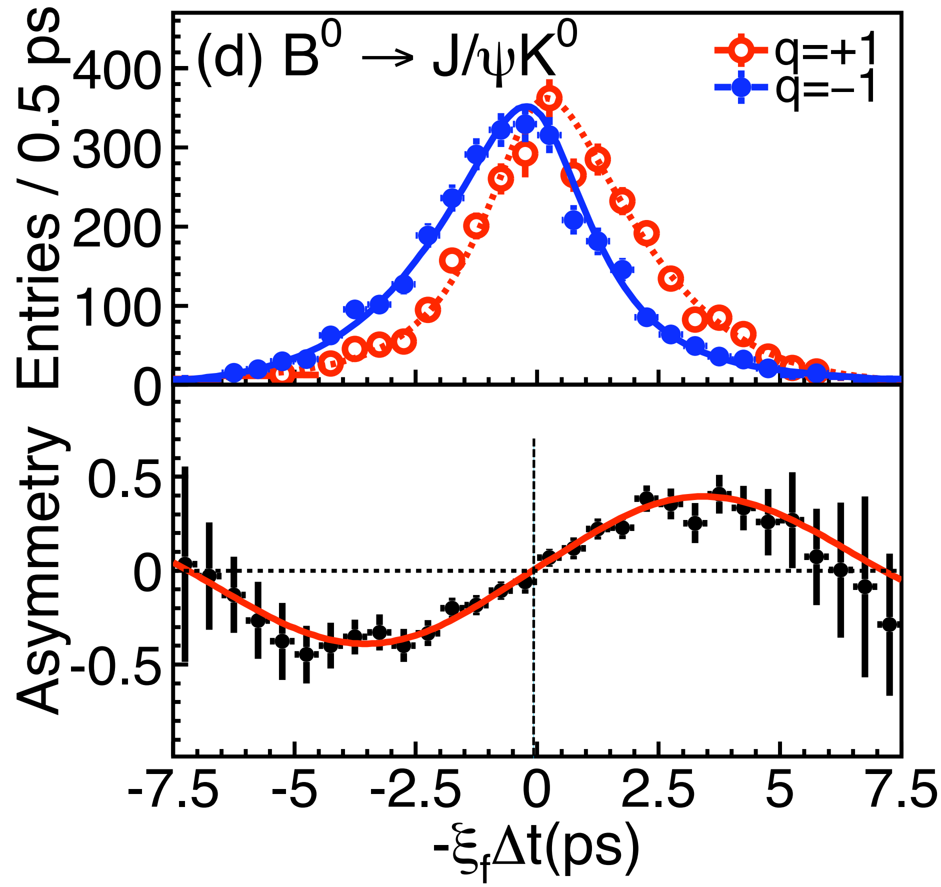
Measurement of ϕ_1 (β) with $B^0 \to J/\psi K^0$:

Aubert et al. (BaBar), PRD 79, 072009 (2009) ***[426 fb^{-1}]***



Chen et al. (Belle), PRL 98, 031802 (2007) ***[492 fb^{-1}]***





Asymmetry = sin(2ϕ_1) sin$\Delta m t$

$\Rightarrow$ sin(2ϕ_1) = 0.666 ± 0.031 ± 0.013

$\mathcal{A}$ = -0.016 ± 0.023 ± 0.018

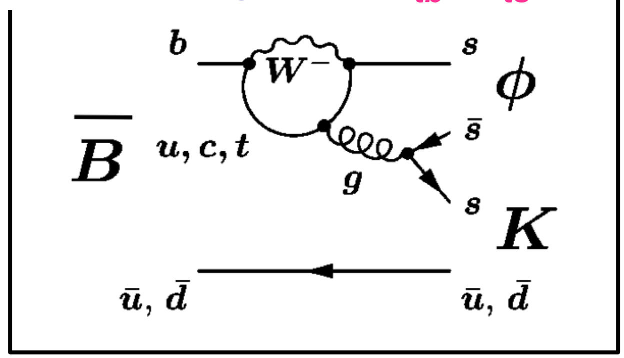
sin(2ϕ_1) = 0.642 ± 0.031 ± 0.017

$\mathcal{A}$ = 0.018 ± 0.021 ± 0.014

ϕ_1 (β) with other final states:

Repeat measurement with other final states, then compare to $B^0 \rightarrow J/\psi\, K^0$: (calculate $\Delta\phi_1$):

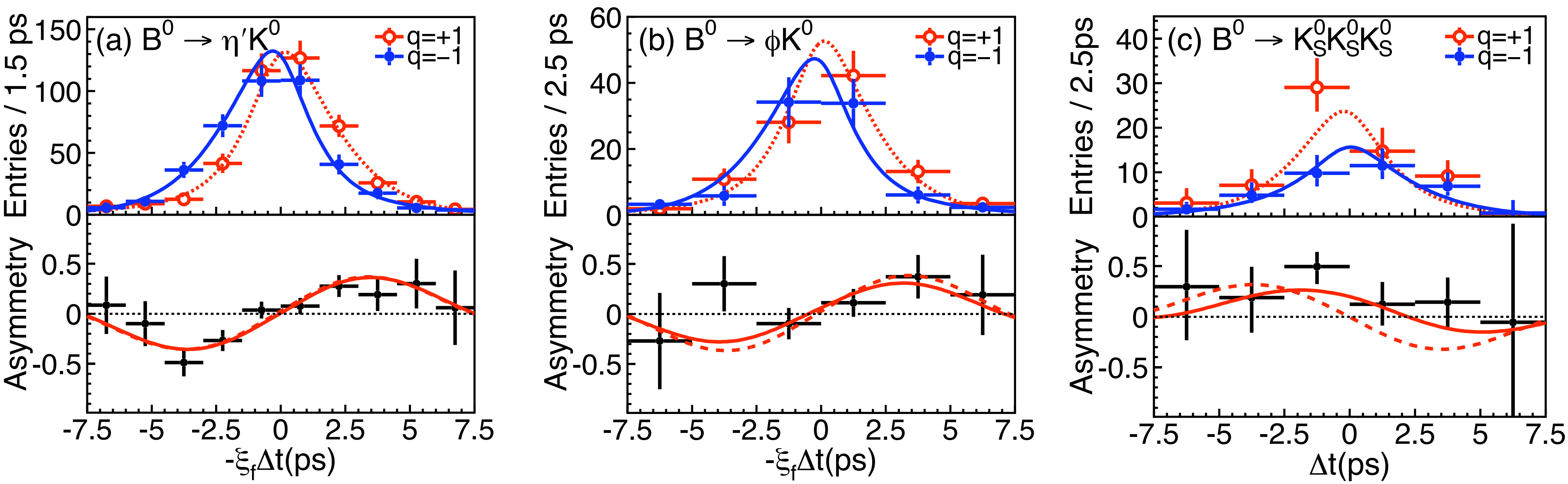
"Loop" diagram: V_{tb}, V_{ts}



e.g., $B^0 \rightarrow \phi\, K^0_S$

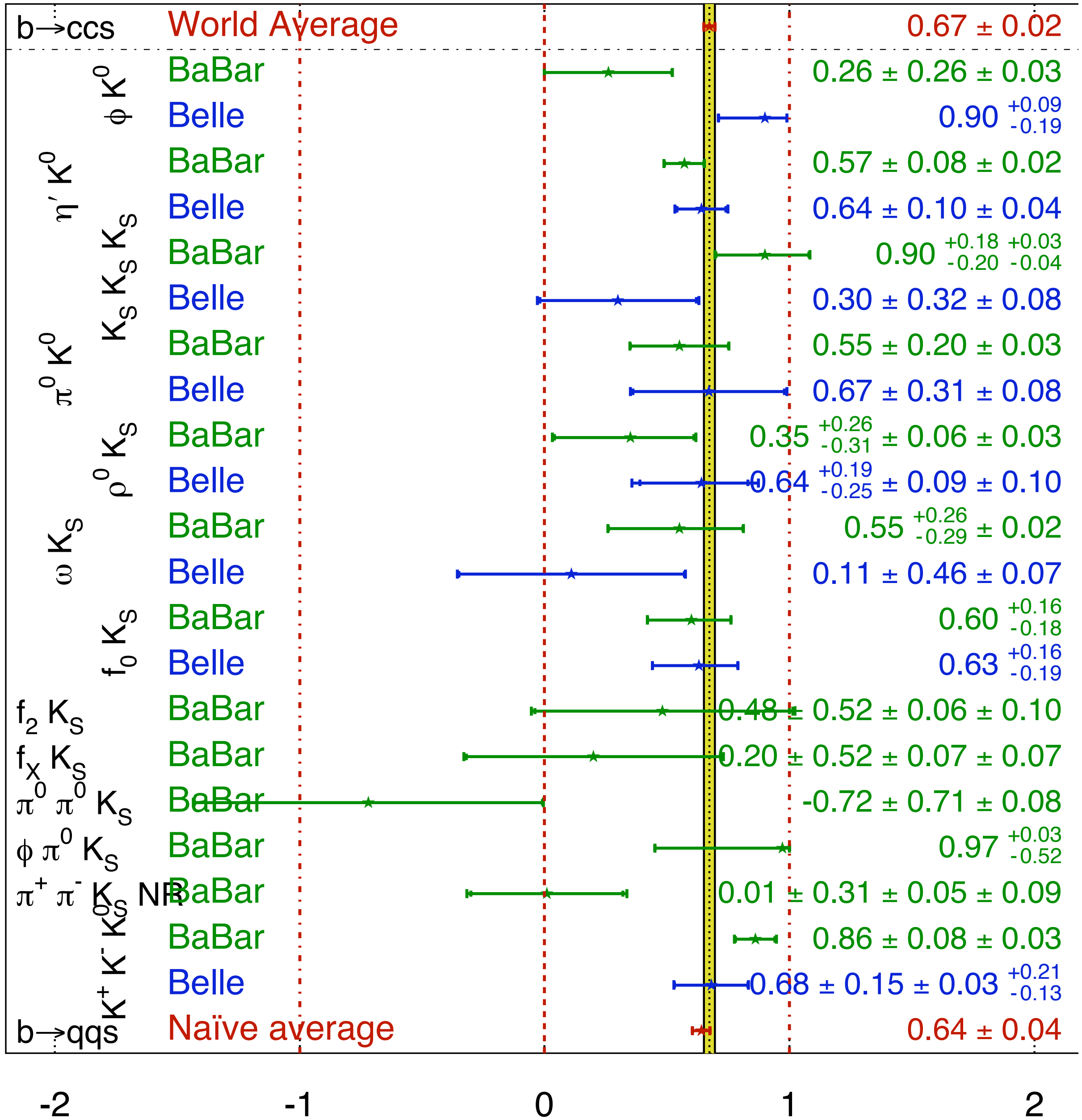
(amplitude proceeds via penguion loop; same weak phase as $J/\psi\, K^0$)

Chen et al. (Belle), PRL 98, 031802 (2007) **[492 fb^{-1}]**



ϕ_1 (β) with other final states

$\sin(2\phi_1^{\mathrm{eff}})$ HFAG FPCP 2010 PRELIMINARY

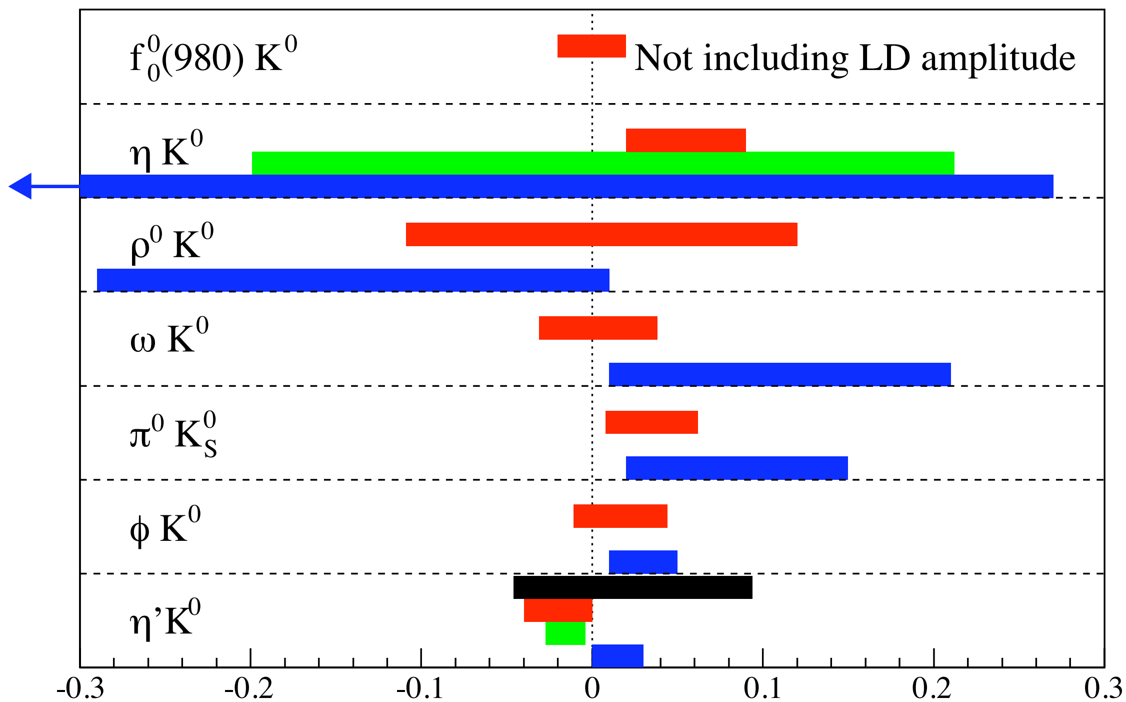


⇒ ***sin(2ϕ_1) values from b→ qqs now appear consistent with values from b→ ccs. Previously, WA appeared lower...***

Measuring mixing phases, cont'd

sin(2ϕ_1) values from b→ qqs appeared lower than values from b→ ccs, but theory predicts slightly higher:

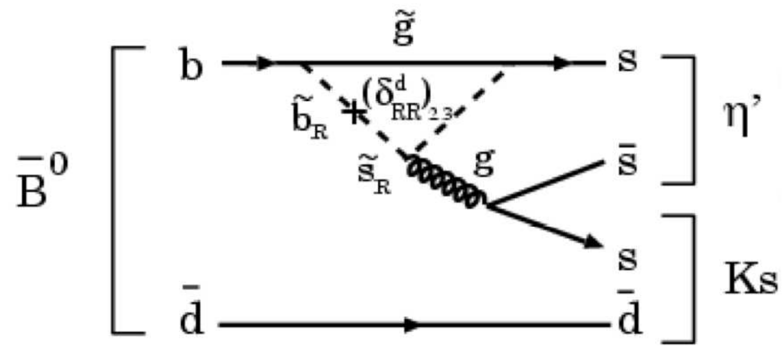
Bevan, arXiv:0812.4388:



$\Delta S = \sin(2\phi_1)_{qqs} - \sin(2\phi_1)_{\psi K0}$

- Blue: *Beneke, PLB 620, 143 (2005), QCDF*
- Green: *Williamson, Zupan, PRD74, 014003 (2006), SCET/QCDF*
- Red: *Cheng et al., PRD72, 014006 (2005), QCDF*
- Black: *Gronau et al., PRD74, 093003 (2006), SU(3)*

could this be sign of new phases from supersymmetry?
(i.e., 41 new phases in MSSM)



A future flavor factory will clarify this (current measurements statistics limited, expect factor of 5-10 improvement in ΔS)

Note: the benchmark value from $B^0 \to J/\psi\ K^0$ will be improved by LHCb

Measurement of $\phi_2\,(\alpha)$ with $B^0 \to \pi^+\pi^-$:

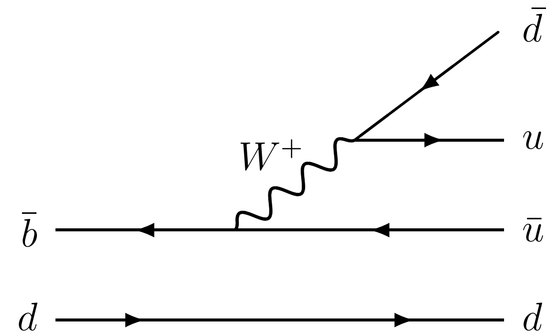
$$\lambda = \sqrt{\frac{M_{12}^*}{M_{12}}}\,\frac{\bar{A}_f}{A_f} = +\left(\frac{V_{td}\,V_{tb}^*}{V_{td}^*\,V_{tb}}\right)\left(\frac{V_{ub}\,V_{ud}^*}{V_{ub}^*\,V_{ud}}\right)$$

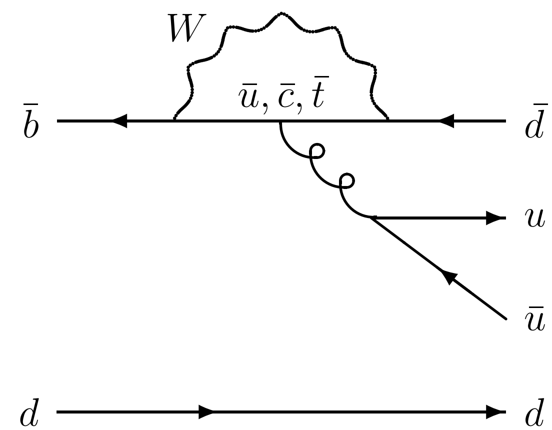
$$= \frac{-V_{tb}^* V_{td}/(V_{ub}^* V_{ud})}{-V_{tb} V_{td}^*/(V_{ub} V_{ud}^*)}$$

$$= \frac{|\mathcal{M}'|e^{i\phi_2}}{|\mathcal{M}'|e^{-i\phi_2}}$$

$$= e^{2i\phi_2}$$

$$\Rightarrow \quad \mathcal{A}_{\pi\pi} = 0 \qquad \mathcal{S}_{\pi\pi} = \sin(2\phi_2)$$



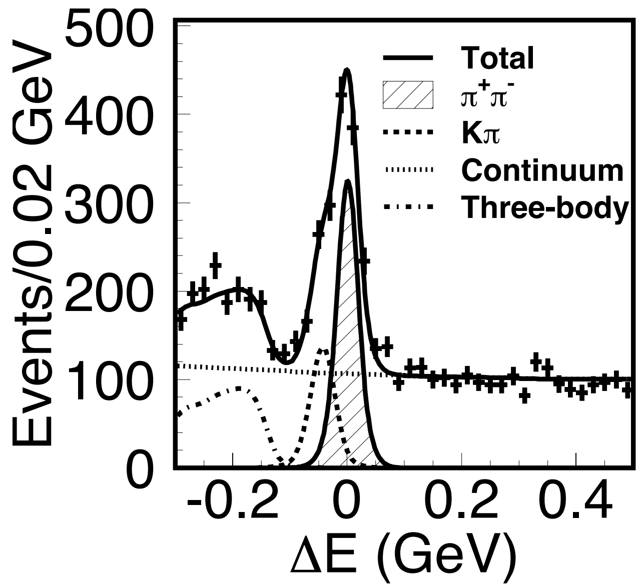


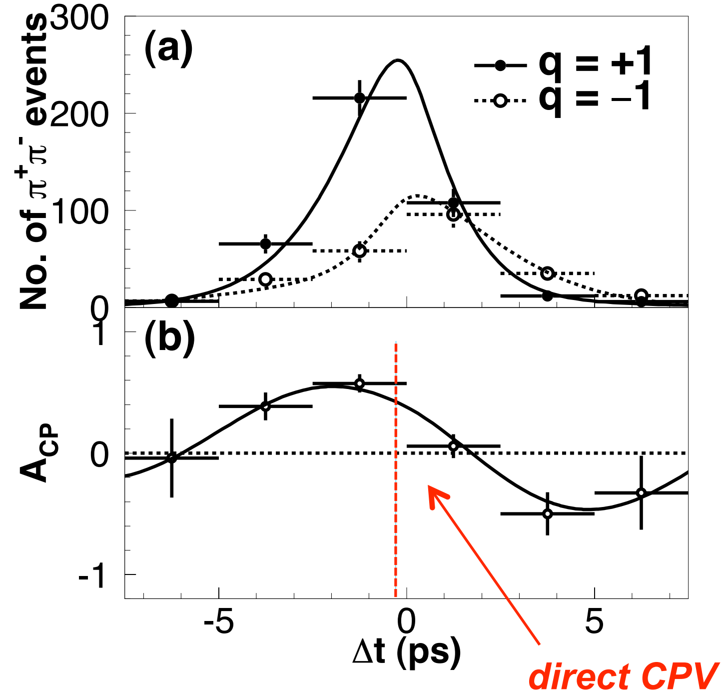
***...if no penguin*. But there is a penguin contribution (with a different weak phase), which breaks these equalities:**

$$\mathcal{S}_{\pi\pi} = \sqrt{(1 - \mathcal{A}^2_{\pi\pi})}\,\sin(2\varphi_2 + \kappa)$$

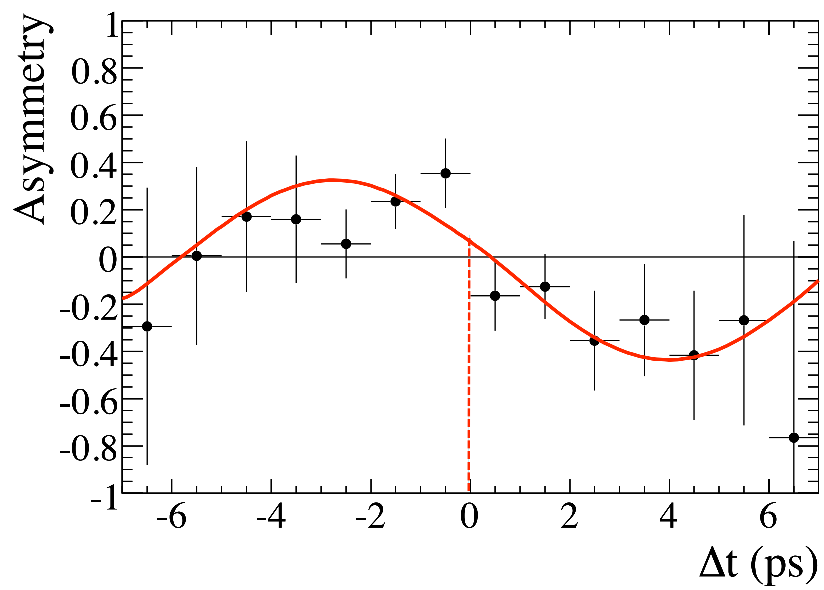
Measurement of $\phi_2\,(\alpha)$ with $B^0 \rightarrow \pi^+\pi^-$:

Ishino et al. (Belle), PRL 98, 211801 (2007) ***[500 fb^{-1}]***





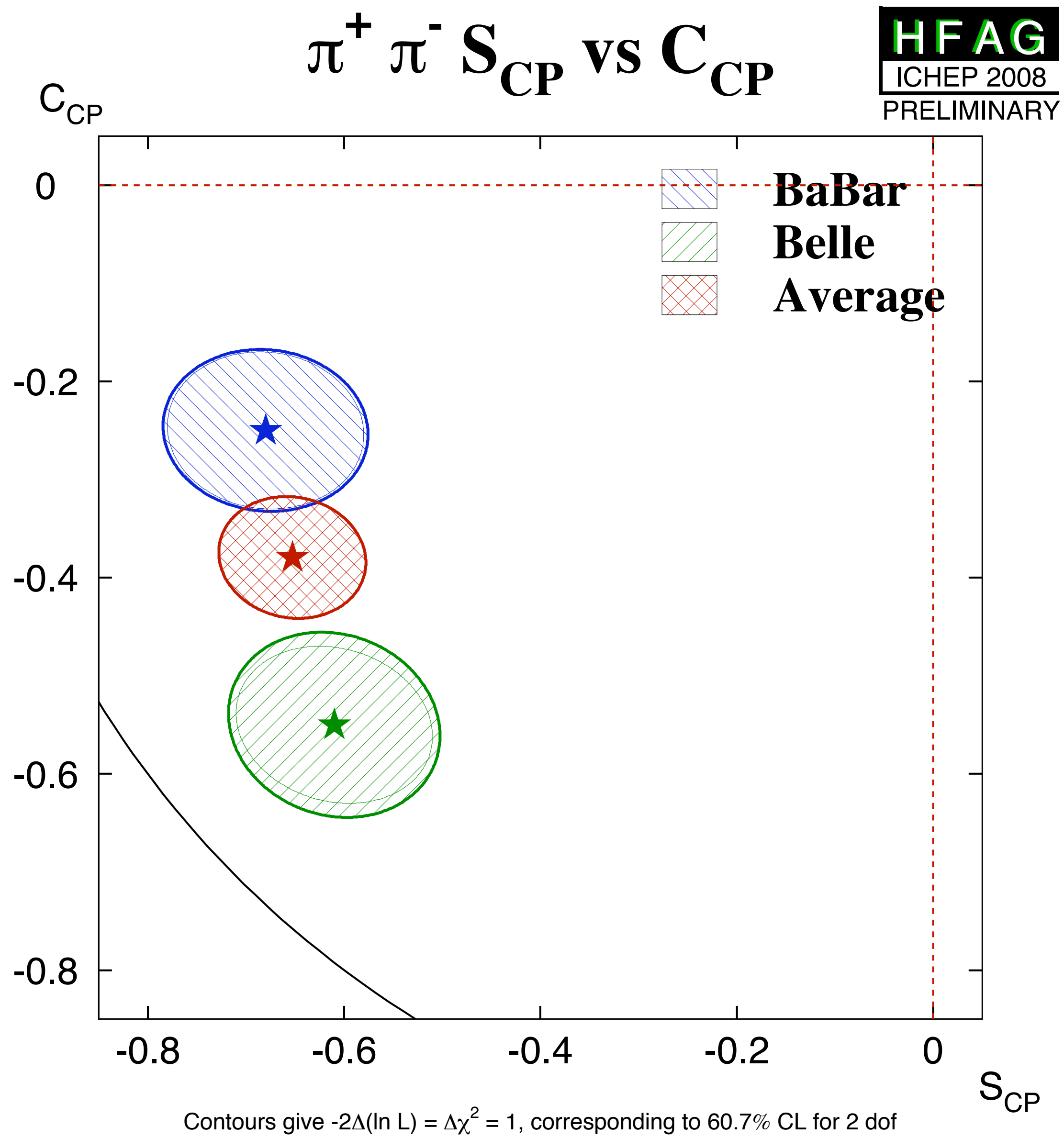
Aubert et al. (BaBar), arXiv:0807.4226 ***[426 fb^{-1}]***



Measurement of ϕ_2 (α) with $B^0 \rightarrow \pi^+\pi^-$:

Ishino et al. (Belle),
PRL 98, 211801 (2007)
[500 fb^{-1}]

Aubert et al. (BaBar),
arXiv:0807.4226
[426 fb^{-1}]



Maximum likelihood fit to Δt ($B^0 \to \pi^+\pi^-$):

$$\mathcal{L}_i = \int \Big[f_{\pi\pi} P_{\pi\pi}(\Delta t') + f_{K\pi} P_{K\pi}(\Delta t') \Big] \cdot R_{hh}(\Delta t_i - \Delta t') + f_{q\bar{q}} P_{q\bar{q}}(\Delta t') \cdot R_{q\bar{q}}(\Delta t_i - \Delta t')\, dt'$$

$$P^{(\ell)}_{B^0\to\pi\pi} = \frac{e^{-|\Delta t|/\tau_B}}{\mathcal{N}} \Big\{ 1 + q(1-2\omega_\ell) \big[\mathcal{A}_{\pi\pi} \cos(\Delta m\, \Delta t) + \mathcal{S}_{\pi\pi} \sin(\Delta m\, \Delta t) \big] \Big\}$$

$$P_{K\pi} = \frac{e^{-|\Delta t|/\tau_B}}{4\tau_B} \Big\{ 1 + q(1-2\omega_\ell) \mathcal{A}^{\rm eff}_{K\pi} \cos(\Delta m\, \Delta t) \Big\} \qquad (\mathcal{A}_{K\pi} = -0.109 \pm 0.019)$$

$$P_{q\bar{q}} = f\, \frac{e^{-|\Delta t|/\tau_{q\bar{q}}}}{2\tau_{q\bar{q}}} + (1-f)\, \delta(\Delta t)\ ,$$

$$f_{\pi\pi} = \frac{F_{\pi\pi}(\Delta E,\, M_{bc}) \cdot f_\ell(\pi\pi)}{[\, F_{\pi\pi}(\Delta E,\, M_{bc}) + F_{K\pi}(\Delta E,\, M_{bc})\,] \cdot f_\ell(\pi\pi) + F_{q\bar{q}}(\Delta E,\, M_{bc}) \cdot f_\ell(q\bar{q})}$$

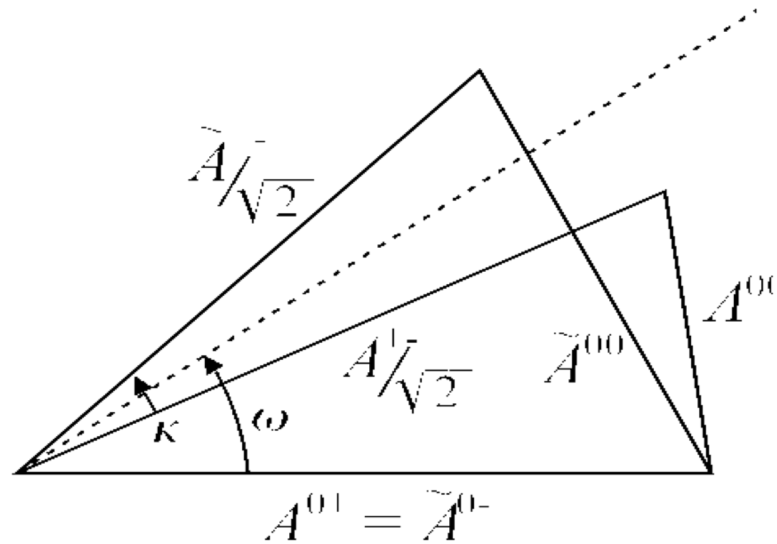
Isospin decomposition for $\phi_2\,(\alpha)$:

Gronau and London,
PRL 65, 3381 (1990)

$$\frac{A(B^0 \to \pi^+\pi^-)}{\sqrt{2}} + A(B^0 \to \pi^0\pi^0) = A(B^+ \to \pi^+\pi^0)$$
$$\frac{A(\overline{B}^0 \to \pi^+\pi^-)}{\sqrt{2}} + A(\overline{B}^0 \to \pi^0\pi^0) = A(B^- \to \pi^-\pi^0)$$

$$\left|A^{+-}_{\rm th}\right| = \sqrt{a^{+-}(1-\mathcal{A}_{\pi\pi})}$$
$$\left|\overline{A}^{+-}_{\rm th}\right| = \sqrt{a^{+-}(1+\mathcal{A}_{\pi\pi})}$$
$$\left|A^{0-}_{\rm th}\right| = \left|A^{0+}_{\rm th}\right| = \sqrt{a^{0+}}$$
$$\left|A^{00}_{\rm th}\right|^2 = \frac{\left|A^{+-}_{\rm th}\right|^2}{2} + \left|A^{0+}_{\rm th}\right|^2 - \sqrt{2}\left|A^{+-}_{\rm th}\right|\left|A^{+0}_{\rm th}\right|\cos(\omega-\kappa/2)$$
$$\left|\overline{A}^{00}_{\rm th}\right|^2 = \frac{\left|\overline{A}^{+-}_{\rm th}\right|^2}{2} + \left|A^{0+}_{\rm th}\right|^2 - \sqrt{2}\left|\overline{A}^{+-}_{\rm th}\right|\left|A^{+0}_{\rm th}\right|\cos(\omega+\kappa/2)$$

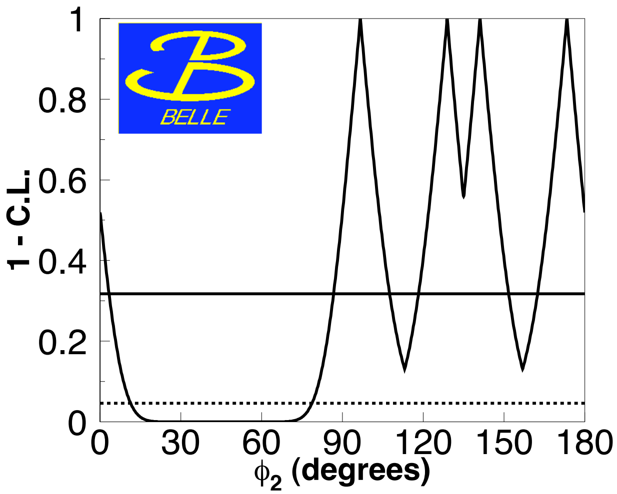
$$B^{\pi^+\pi^-}_{\rm th} = \left(\left|A^{+-}_{\rm th}\right|^2 + \left|\overline{A}^{+-}_{\rm th}\right|^2\right)/2 = a^{+-}$$
$$B^{\pi^0\pi^0}_{\rm th} = \left(\left|A^{00}_{\rm th}\right|^2 + \left|\overline{A}^{00}_{\rm th}\right|^2\right)/2$$
$$B^{\pi^0\pi^+}_{\rm th} = \left|A^{0+}_{\rm th}\right|^2(\tau_{B^\pm}/\tau_{B^0}) = a^{+0}\cdot(\tau_{B^\pm}/\tau_{B^0})$$
$$\mathcal{A}^{\pi^0\pi^0}_{\rm th} = \frac{\left|\overline{A}^{00}_{\rm th}\right|^2 - \left|A^{00}_{\rm th}\right|^2}{\left|\overline{A}^{00}_{\rm th}\right|^2 + \left|A^{00}_{\rm th}\right|^2}$$
$$\mathcal{A}^{\pi^+\pi^-}_{\rm th} = \mathcal{A}_{\pi\pi}$$
$$\mathcal{S}^{\pi^+\pi^-}_{\rm th} = \sqrt{1-\mathcal{A}'^2_{\pi\pi}}\ \sin(2\phi_2+\kappa)$$

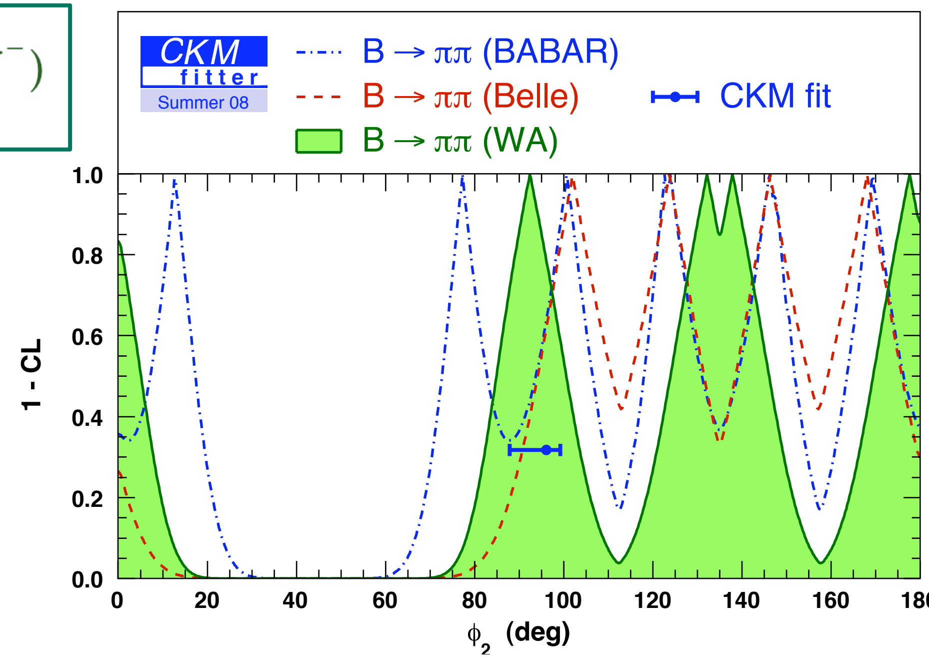


6 parameters + 6 observables
⇒ all determined

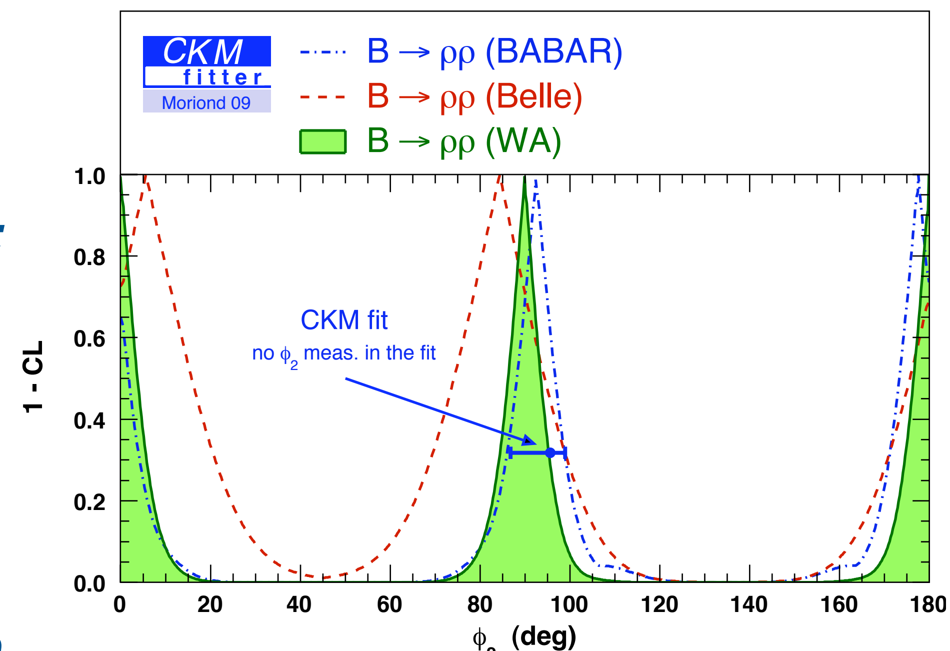
Isospin analysis for $\phi_2\,(\alpha)$:

$$\chi^2(\vec{y}) \;=\; \sum \frac{(x_{\rm exp} - x_{\rm th})^2}{\sigma^2_{\rm exp}} \;+\; \chi^2_{FC}\,(\mathcal{A}^{\pi^+\pi^-}_{\rm th},\, \mathcal{S}^{\pi^+\pi^-}_{\rm th})$$





Same isospin analysis for $B^0 \to \rho\rho$ system:



Relation among ϕ_2, δ, and $|P/T|$:

Gronau and Rosner, PRD 65, 093012 (2002)

$$A(B^0\to\pi^+\pi^-) = -\left(|T|e^{i\delta_T}e^{i\phi_3} + |P|e^{i\delta_P}\right)$$
$$A(\overline{B}^0\to\pi^+\pi^-) = -\left(|T|e^{i\delta_T}e^{-i\phi_3} + |P|e^{i\delta_P}\right)$$

$$\Rightarrow \quad \lambda_{\pi\pi} \equiv \frac{q}{p}\frac{\overline{A}_{\pi\pi}}{A_{\pi\pi}} = e^{i\phi_2}\frac{1+|P/T|\,e^{i(\delta+\phi_3)}}{1+|P/T|\,e^{i(\delta-\phi_3)}}$$

$$(\delta \equiv \delta_P - \delta_T)$$

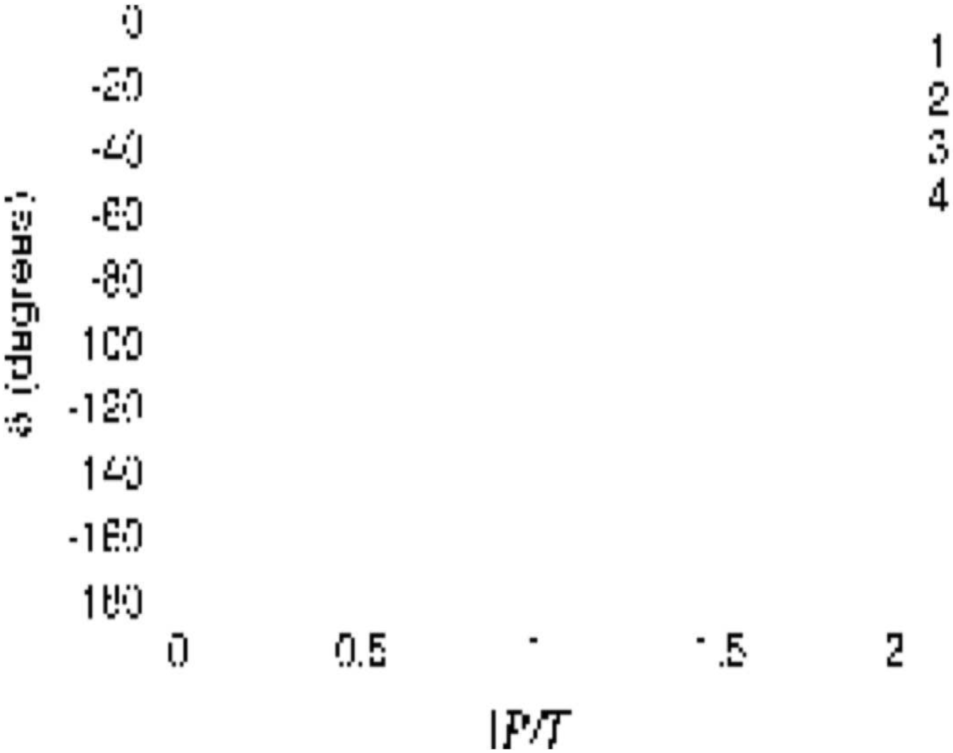
Take ϕ_1 as measured in $B^0\to J/\psi\, K^0$ decays

$\Rightarrow$ 2 constraints, 3 unknowns

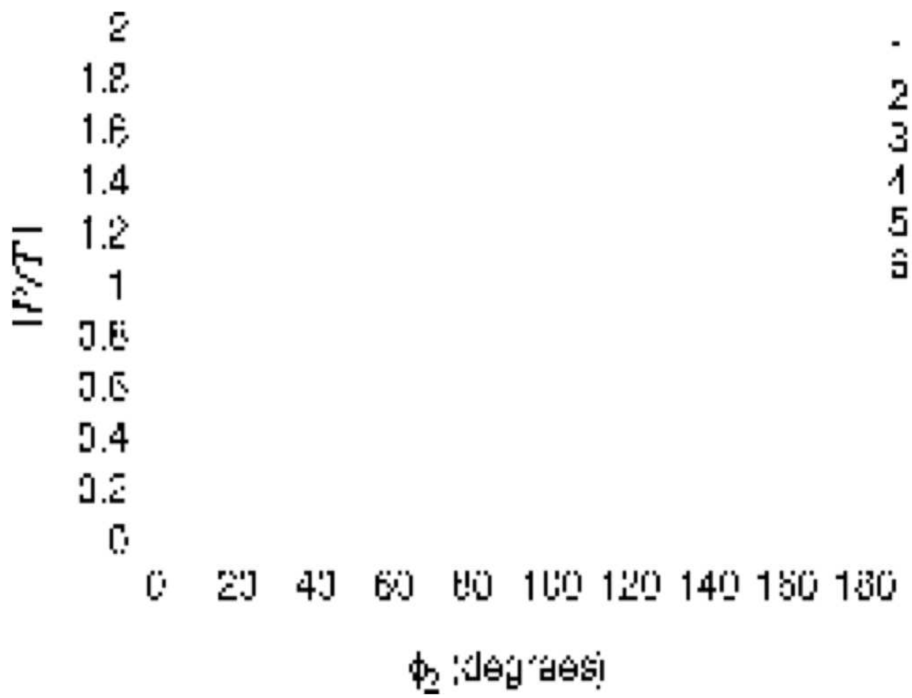
$$A_{\pi\pi} \equiv \frac{|\lambda|^2-1}{|\lambda|^2+1} = \frac{-2|P/T|\sin(\phi_1+\phi_2)\sin\delta}{1-2|P/T|\cos(\phi_1+\phi_2)\cos\delta+|P/T|^2}$$

$$S_{\pi\pi} \equiv \frac{2Im\lambda}{|\lambda|^2+1}$$
$$= \frac{2|P/T|\sin(\phi_1-\phi_2)\cos\delta+\sin 2\phi_2-|P/T|^2\sin 2\phi_1}{1-2|P/T|\cos(\phi_1+\phi_2)\cos\delta+|P/T|^2}$$

Constraints among ϕ_2, δ, and |P/T|:

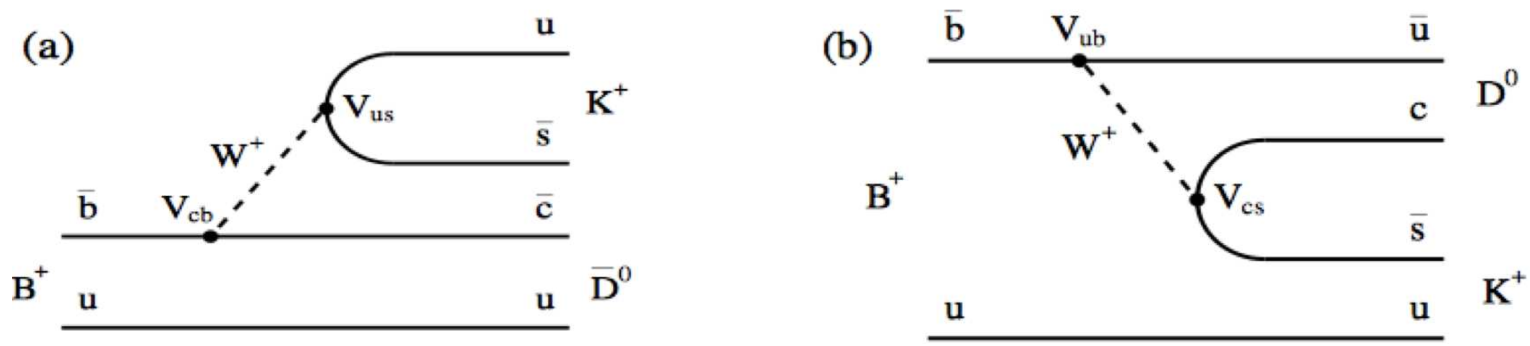


For any |*P/T*|
$\delta < -4^o$ (95% CL)
For any δ
|*P/T*| > 0.17 (95% CL)



For |*P/T*|=0.6 (for example)
$72^o < \phi_2 < 146^o$ (95% CL)

Measurements of $\phi_3\,(\gamma)$:



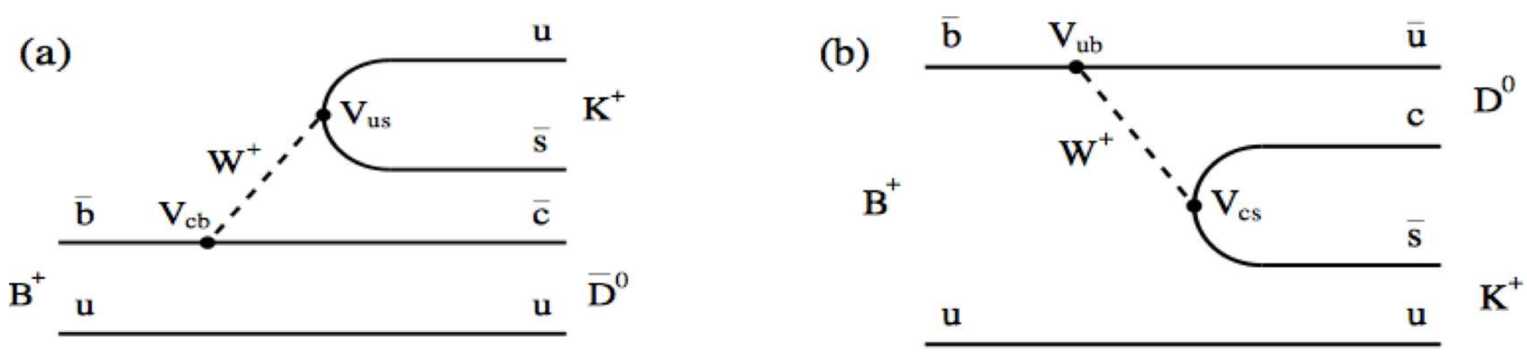
Relative weak phase = ***Arg*** $[V_{ub}\, V_{cs}^* / (V_{cb} V_{us}^*)] \cong \gamma$

Final states f in which both $D^0 \to f$ and $\bar{D}^0 \to f$ are possible will have both amplitudes contribute: the subsequent interference causes CPV.

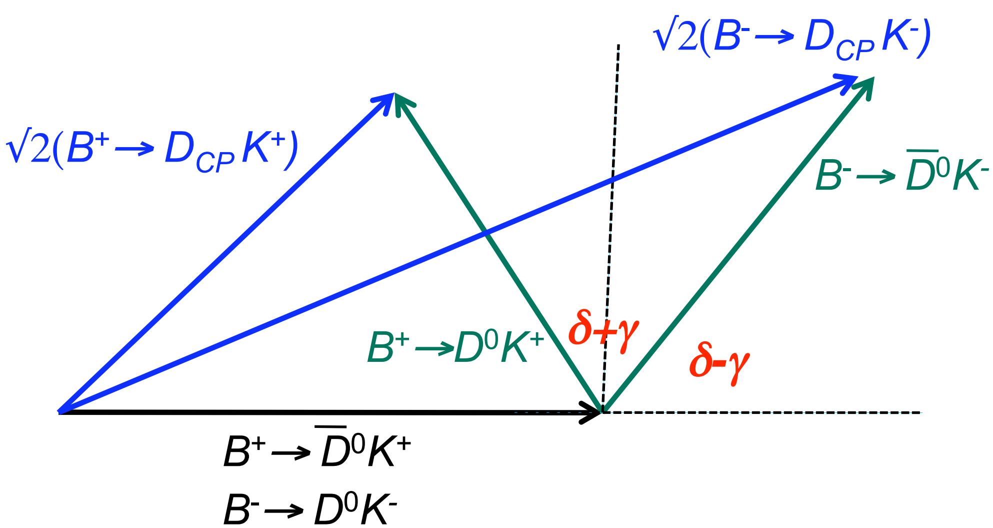
1) GLW method: use CP eigenstates
2) ADS method: use CF and DCS
3) Dalitz plot analysis: use 3-body

Measurements of ϕ_3 : Gronau-London-Wyler (GLW)

Gronau and London, PLB 253, 483 (1991); Gronau and Wyler, PLB 265, 172 (1991)



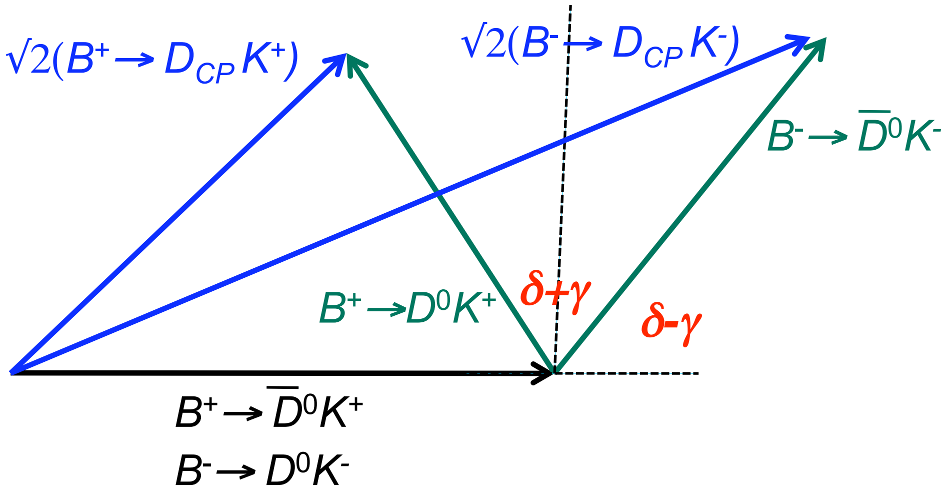
phase difference = $\gamma+\delta$ *(B+) or* $\gamma-\delta$ *(B-)*

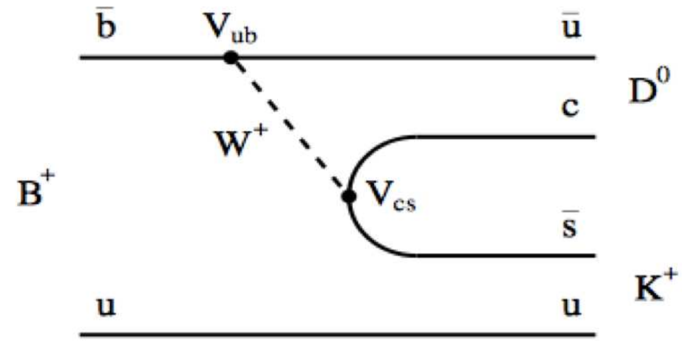


Method:

measure 6 rates
reconstruct 2 triangles
extract γ (ϕ_3)

Measurements of ϕ_3: Gronau-London-Wyler (GLW)





Problem: *the V_{ub}-suppressed + color-suppressed decays $B^+ \to D^0 K^+$, $B^- \to \overline{D}^0 K^-$ have too small a rate to be well-measured at a B factory.*

Solution: *from two triangles we have (cosine rule):*

Gronau, PRD 58, 037301 (1998)

$$
\begin{aligned}
2\Gamma(B^+ \to D_1 K^+) &= \overline{A}^2 + A^2 + 2A\overline{A}\cos(\delta + \gamma) \\
2\Gamma(B^+ \to D_2 K^+) &= \overline{A}^2 + A^2 - 2A\overline{A}\cos(\delta + \gamma)
\end{aligned}
$$

$$
\left(\frac{\overline{A}}{A} = \frac{|\mathcal{A}(B^- \to \overline{D}^0 K^-)|}{|\mathcal{A}(B^- \to D^0 K^-)|} \equiv r \right)
$$

$\Rightarrow$
$$
\boxed{\Gamma(B^+ \to D_1 K^+) + \Gamma(B^+ \to D_2 K^+) = \Gamma(B^+ \to D^0 K^+) + \Gamma(B^+ \to \overline{D}^0 K^+)}
$$

$\Rightarrow$ ***this relationship eliminates the need to measure the small rate of $\Gamma(B^+ \to D^0 K^+)$***

Measurements of ϕ_3 : Gronau-London-Wyler (GLW)

Define new observables: *Gronau, PRD 58, 037301 (1998)*

$$\mathcal{R}_1 \equiv \frac{2\left[B(B^-\to D_1 K^-) + B(B^+\to D_1 K^+)\right]}{B(B^-\to D^0 K^-) + B(B^+\to \overline{D}{}^0 K^+)} = 1 + r^2 + 2r\cos\delta\cos\phi_3$$

$$\mathcal{R}_2 \equiv \frac{2\left[B(B^-\to D_2 K^-) + B(B^+\to D_2 K^+)\right]}{B(B^-\to D^0 K^-) + B(B^+\to \overline{D}{}^0 K^+)} = 1 + r^2 - 2r\cos\delta\cos\phi_3$$

$$\mathcal{A}_1 \equiv \frac{B(B^-\to D_1 K^-) - B(B^+\to D_1 K^+)}{B(B^-\to D_1 K^-) + B(B^+\to D_1 K^+)} = \frac{2r\sin\delta\sin\phi_3}{\mathcal{R}_1}$$

$$\mathcal{A}_2 \equiv \frac{B(B^-\to D_2 K^-) - B(B^+\to D_2 K^+)}{B(B^-\to D_2 K^-) + B(B^+\to D_2 K^+)} = \frac{-2r\sin\delta\sin\phi_3}{\mathcal{R}_2}$$

$\Longrightarrow$ ***4 observables, 3 unknowns (r, δ, ϕ_3), solved***

Last problem: *the ratios $\mathcal{R}_1$ and $\mathcal{R}_2$ depend on D_1, D_2, D^0, and $\overline{D}{}^0$ branching fractions to the final states used; some of these have notable uncertainty.*

Solution: *define 2 more observables:*
Gronau, PLB 557, 198 (2003)

$$R^{D_{1,2}} \equiv \frac{B(B^-\to D_{1,2} K^-) + B(B^+\to D_{1,2} K^+)}{B(B^-\to D_{1,2} \pi^-) + B(B^+\to D_{1,2} \pi^+)}$$

$$R^{D^0} \equiv \frac{B(B^-\to D^0 K^-) + B(B^+\to D^0 K^+)}{B(B^-\to D^0 \pi^-) + B(B^+\to D^0 \pi^+)}$$

$$\Longrightarrow \quad \mathcal{R}_{1,2} \approx R^{D_{1,2}}/R^{D^0}$$

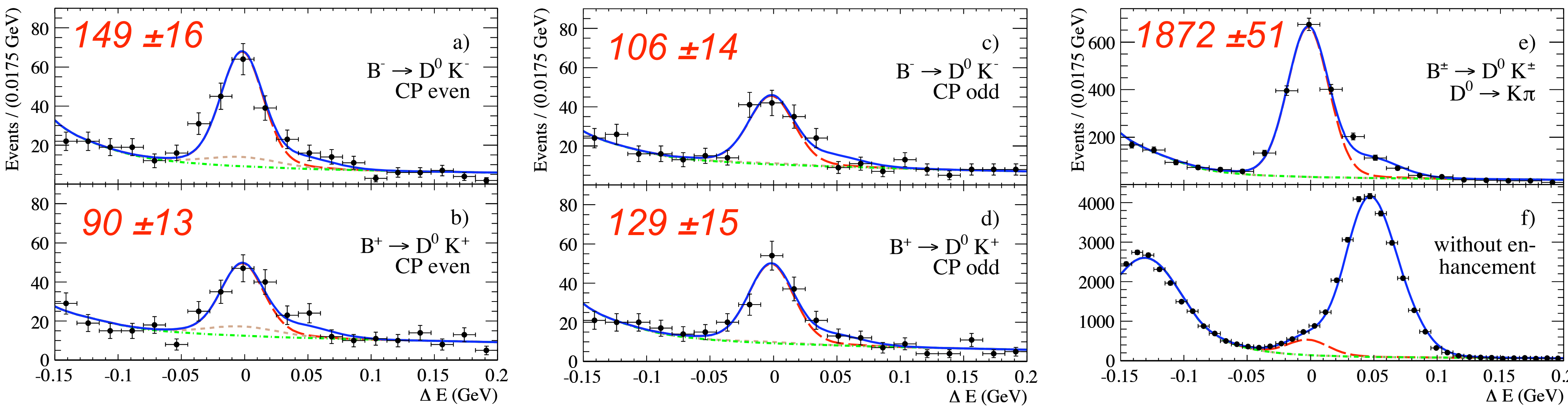
Measurement of ϕ_3 via GLW:

Aubert et al., PRD 77, 111102(R) (2008) **[348 fb^{-1}]**

CP even: $D^0 \to K^+K^-, \pi^+\pi^-$

CP odd: $D^0 \to K_S\pi^0, K_S\omega$

flavor eigenstate: $D^0 \to K^-\pi^+$



$R_{CP+} = 1.06 \pm 0.10 \pm 0.05$

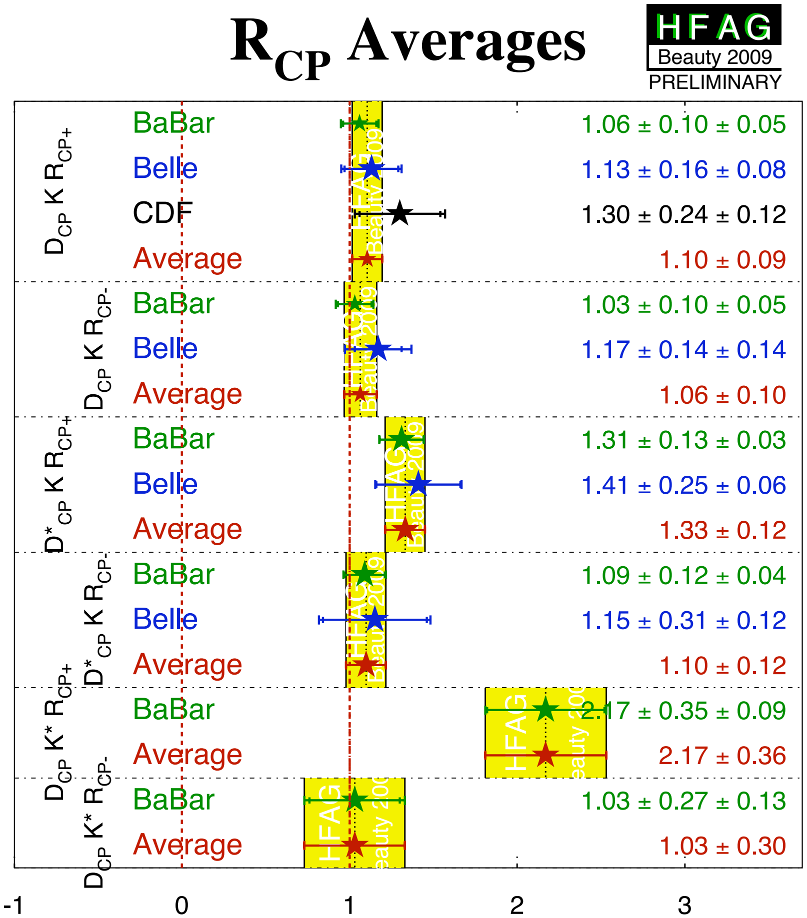
$A_{CP+} = 0.27 \pm 0.09 \pm 0.04$

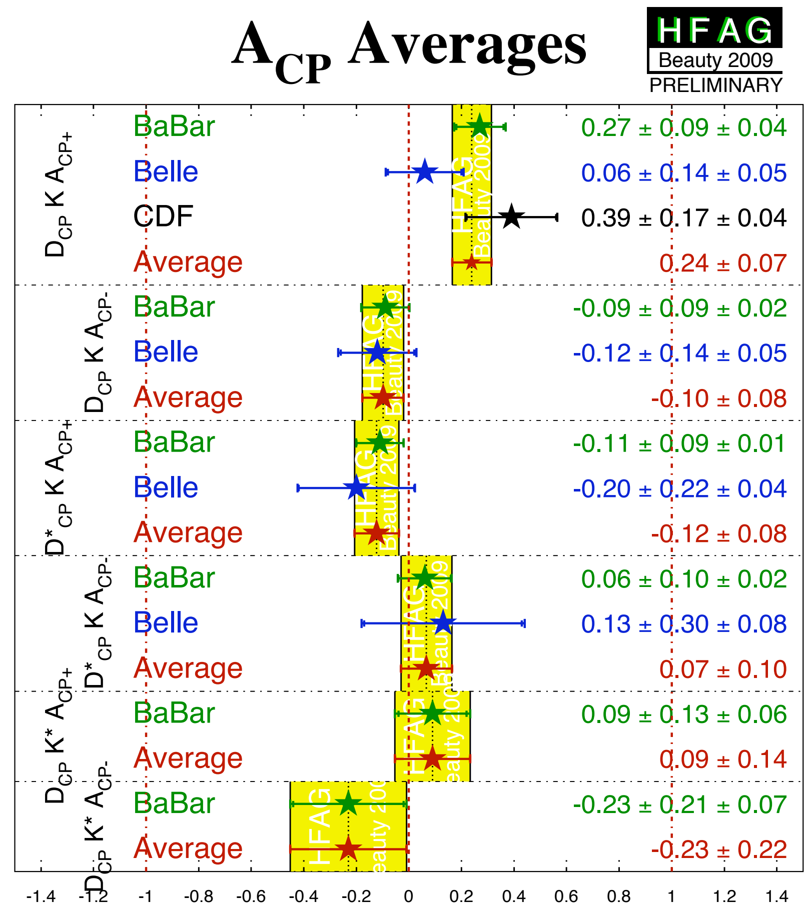
$R_{CP-} = 1.03 \pm 0.10 \pm 0.05$

$A_{CP-} = -0.09 \pm 0.09 \pm 0.02$

Measurement of ϕ_3 via GLW:

Include Belle results *[250 fb-1, PRD 73, 051106 (2006)]* ***and also $B \to D^* K$, $D K^*$:***

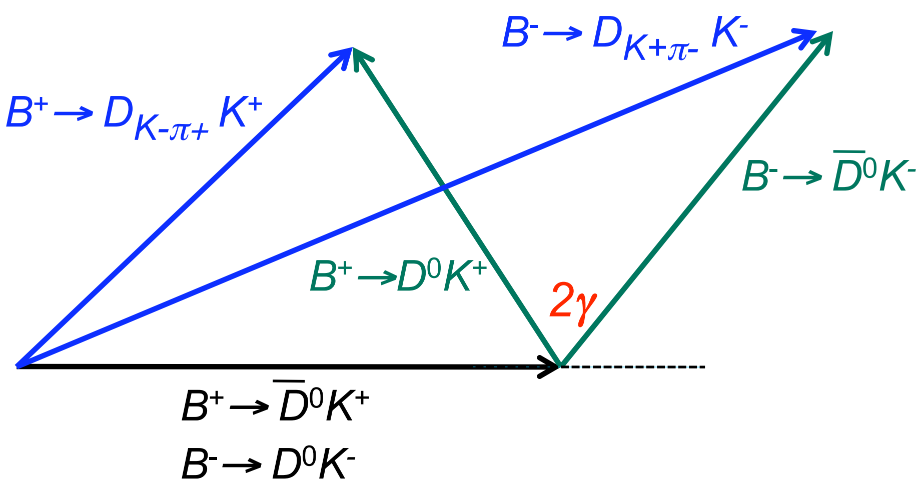




Measurements of ϕ_3: Atwood-Dunietz-Soni (ADS)

Atwood, Dunietz, and Soni, PRL 78, 3257 (1997); PRD 63, 036005 (2001)

For the "common" mode (reachable from both D^0 and $\overline{D}^0$), use a final state f such that $D^0 \to f$ is Cabibbo-favored (CF) and $\overline{D}^0 \to f$ is doubly-Cabibbo-suppressed (DCS). Then interference between two competing amplitudes can be larger than in GLW method



$$r^2 = \left| \frac{A(B^- \to D^0 K^- \to [K^+\pi^-]K^-)}{A(B^- \to \overline{D}^0 K^- \to [K^+\pi^-]K^-)} \right|^2$$

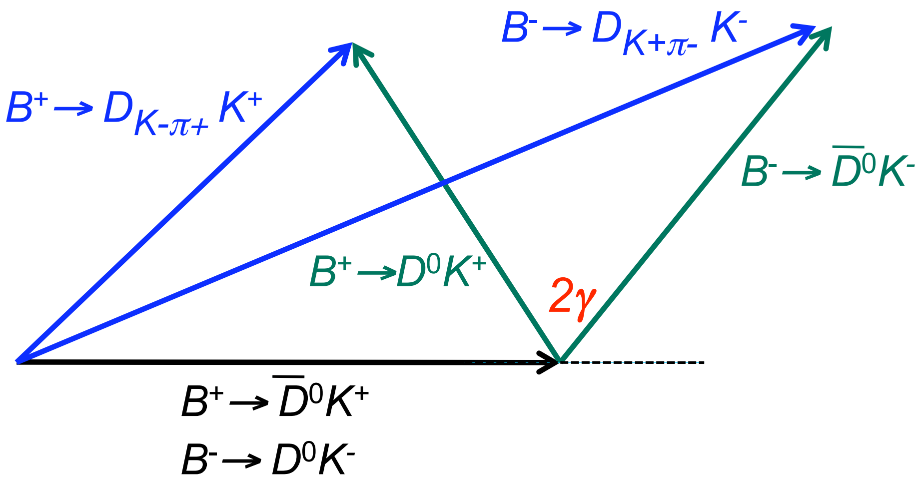
$$\approx \left| \frac{V_{cb}V_{us}^*}{V_{ub}V_{cs}^*} \right|^2 \left| \frac{a_1}{a_2} \right|^2 \frac{B(D^0 \to K^+\pi^-)}{B(\overline{D}^0 \to K^+\pi^-)}$$

$$\approx \left(\frac{1}{0.10}\right)^2 \left(\frac{3}{1}\right)^2 (0.33\%) \approx 3$$

color suppression

Measurements of ϕ_3 : Atwood-Dunietz-Soni (ADS)

Expand GLW formalism to apply to non-CP common states:



$$\Gamma(B^- \to D_{K^+\pi^-}K^-) = \overline{A}^2C^2 + A^2\overline{C}^2 + 2A\overline{A}C\overline{C}\cos(\delta_b + \delta_c + \gamma)$$
$$\Gamma(B^+ \to D_{K^-\pi^+}K^+) = \overline{A}^2C^2 + A^2\overline{C}^2 + 2A\overline{A}C\overline{C}\cos(\delta_b + \delta_c - \gamma)$$

$$\text{Note:} \quad \Gamma(B^- \to D_{K^-\pi^+}K^-) \approx \Gamma(B^+ \to D_{K^+\pi^-}K^+) \approx A^2C^2$$

$$\left(\frac{\overline{A}}{A} = \frac{|\mathcal{A}(B^- \to \overline{D}{}^0K^-)|}{|\mathcal{A}(B^- \to D^0K^-)|} \equiv r_B \qquad \frac{\overline{C}}{C} = \frac{|\mathcal{A}(\overline{D}{}^0 \to K^-\pi^+)|}{|\mathcal{A}(D^0 \to K^-\pi^+)|} \equiv r_D \right)$$

Measurements of ϕ_3 : Atwood-Dunietz-Soni (ADS)

$$\Gamma(B^- \to D_{K^+\pi^-} K^-) \;=\; \overline{A}^2 C^2 + A^2 \overline{C}^2 + 2A\overline{A}C\overline{C}\cos(\delta_b + \delta_c + \gamma)$$
$$\Gamma(B^+ \to D_{K^-\pi^+} K^+) \;=\; \overline{A}^2 C^2 + A^2 \overline{C}^2 + 2A\overline{A}C\overline{C}\cos(\delta_b + \delta_c - \gamma)$$

$$\left(\frac{\overline{A}}{A} = \frac{|\mathcal{A}(B^- \to \overline{D}^0 K^-)|}{|\mathcal{A}(B^- \to D^0 K^-)|} \equiv r_B \qquad \frac{\overline{C}}{C} = \frac{|\mathcal{A}(\overline{D}^0 \to K^-\pi^+)|}{|\mathcal{A}(D^0 \to K^-\pi^+)|} \equiv r_D \right)$$

Define new observables:

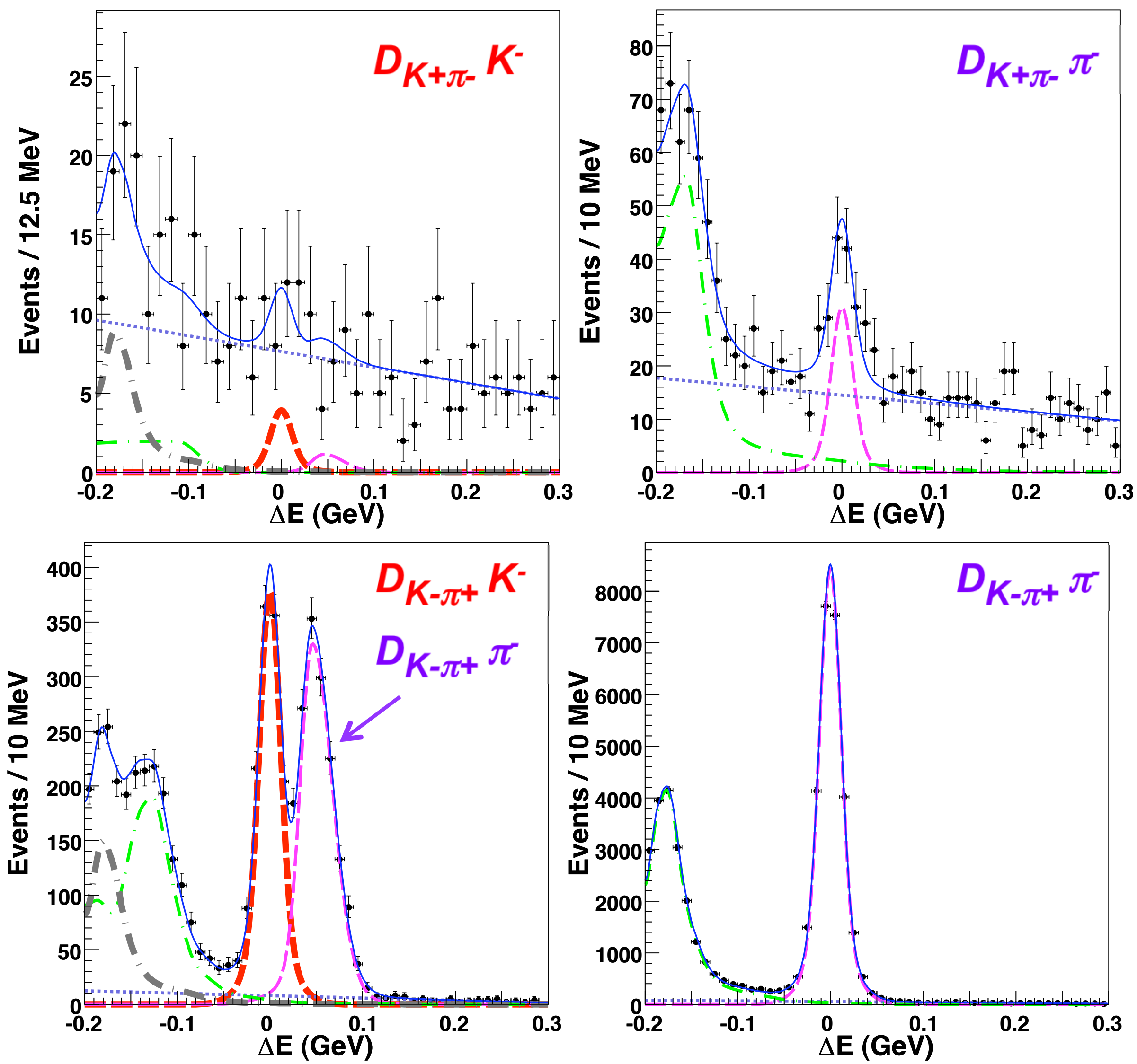
$$\mathcal{R}_{\rm ADS} \equiv \frac{B(B^- \to D_{K^+\pi^-} K^-) + B(B^+ \to D_{K^-\pi^+} K^+)}{B(B^- \to D_{K^-\pi^+} K^-) + B(B^+ \to D_{K^+\pi^-} K^+)}$$
$$= r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_b + \delta_c)\cos\phi_3$$

$$\mathcal{A}_{\rm ADS} \equiv \frac{B(B^- \to D_{K^+\pi^-} K^-) - B(B^+ \to D_{K^-\pi^+} K^+)}{B(B^- \to D_{K^+\pi^-} K^-) + B(B^+ \to D_{K^-\pi^+} K^+)} = \frac{2r_B r_D \sin(\delta_b + \delta_c)\sin\phi_3}{\mathcal{R}_{\rm ADS}}$$

⇒ 2 observables, 3 B-related unknowns (r_B, δ_b, ϕ_3), need to use external information (in addition to external r_D, δ_c)

Measurement of ϕ_3 via ADS:

Horii et al. (Belle), PRD 78, 071901(R) (2008) **[605 fb^{-1}]**



For B→ $D_{K+\pi-}$ K^- : no significant signal (1.3σ excess)

(rate is too low, the original problem of GLW analysis)

$$\mathcal{A}_{\mathrm{ADS}} = -0.1\,^{+0.8}_{-1.0} \pm 0.4$$

(no constraint)

$$\mathcal{R}_{\mathrm{ADS}} = \left(0.78\,^{+0.62}_{-0.57}\,^{+0.20}_{-0.28}\right)\%$$

$$= r_B^2 + r_D^2 + 2 r_B r_D \cos(\delta_b + \delta_c)\cos\phi_3$$

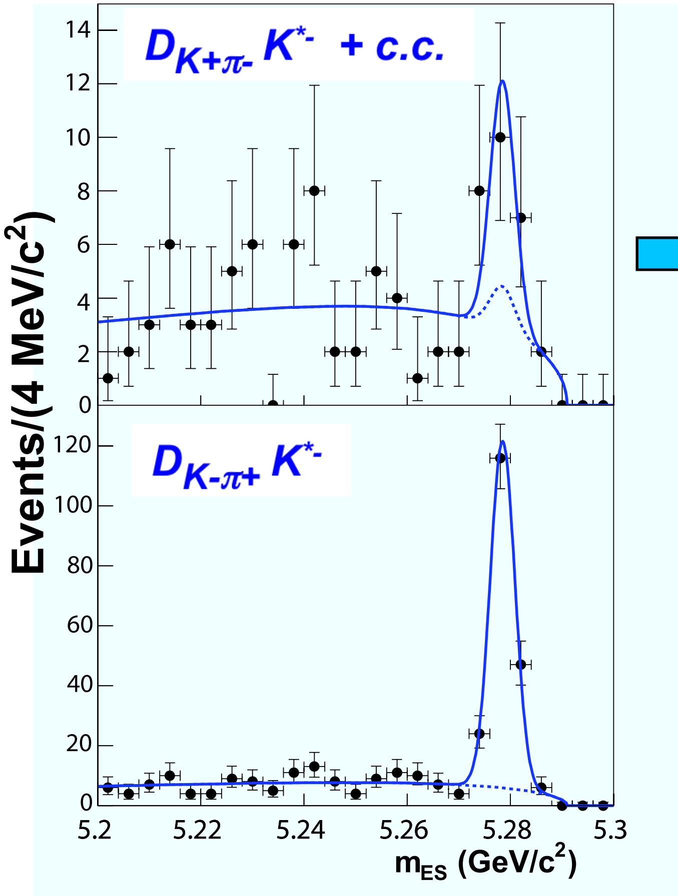
⇒ r_B < 0.19

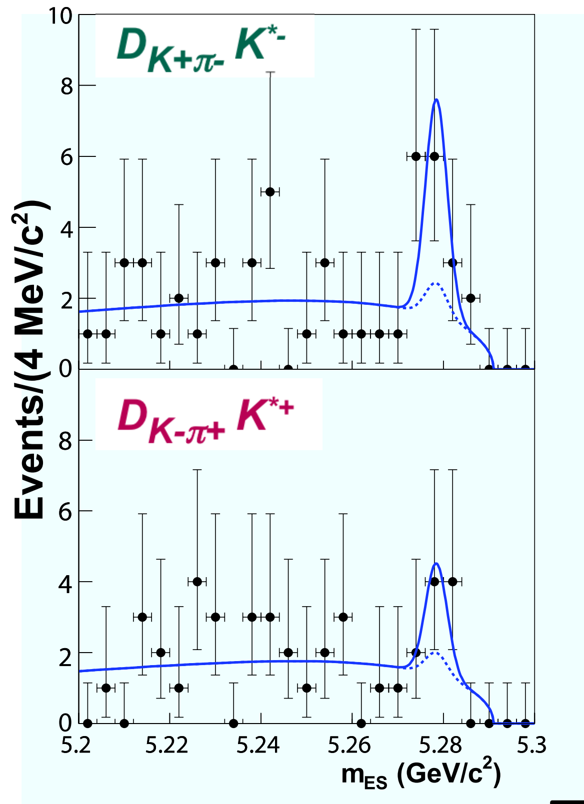
[cos(δ_b+δ_c)cosϕ_3 = -1, 90% CL]

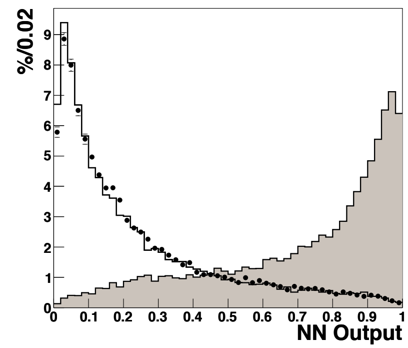
Measurement of ϕ_3 via ADS:

Aubert et al. (BaBar), PRD 80, 092001 (2009) ***[345 fb^{-1}]*** **:**

to increase statistics, use neural net and $B^- \to D K^{*-}$ decays, where $K^{*-} \to K_S \pi^-$







For $B \to D_{K+\pi-} K^{*-}$, signal is visible (fit yield: 11.5 ±5.3 events). Rough asymmetry measurement is possible.

$$\mathcal{A}_{\rm ADS} = -0.34 \pm 0.43 \pm 0.16$$

(consistent with zero)

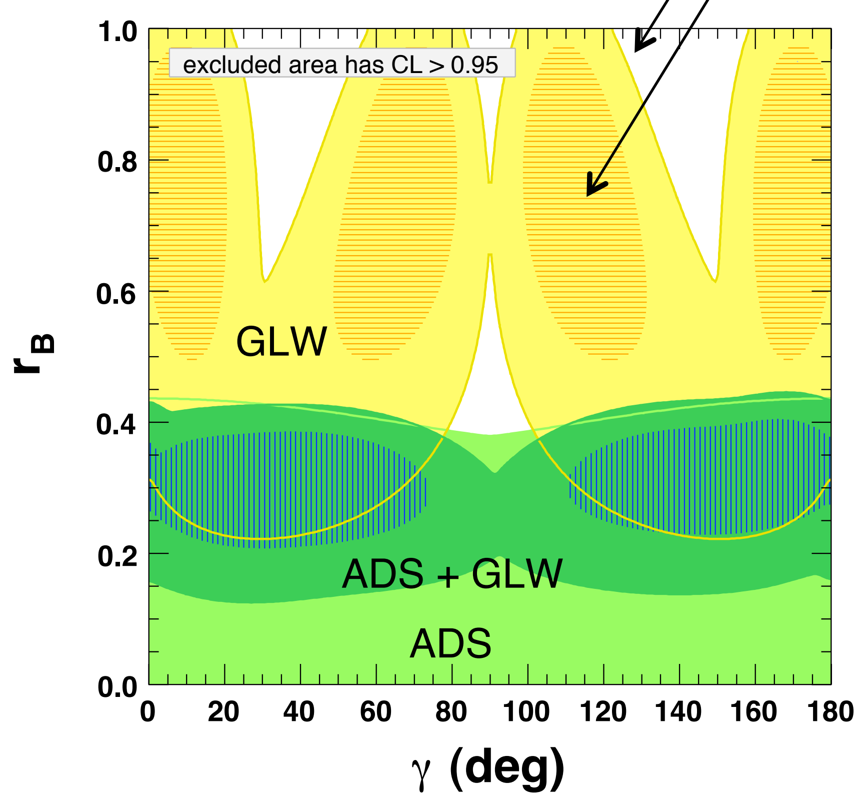
$$\mathcal{R}_{\rm ADS} = (6.6 \pm 3.1 \pm 1.0)\,\%$$

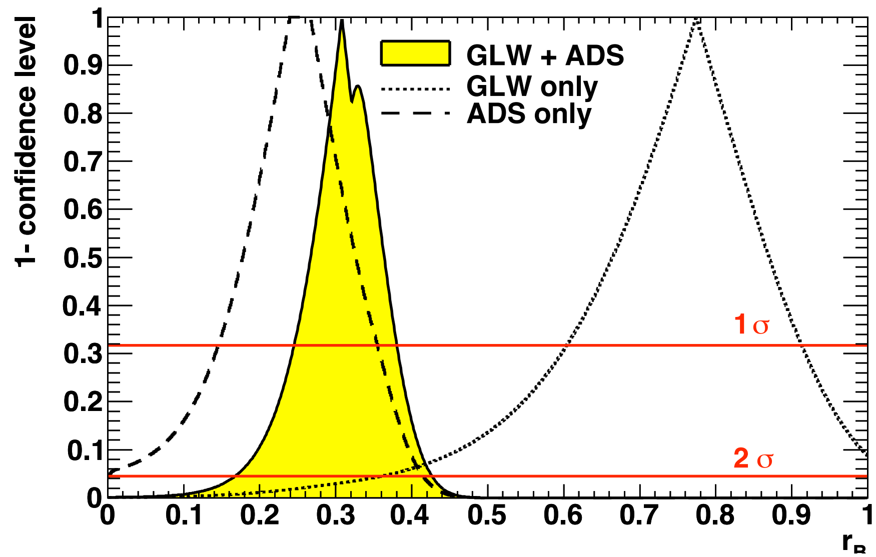
Measurement of ϕ_3 via ADS:

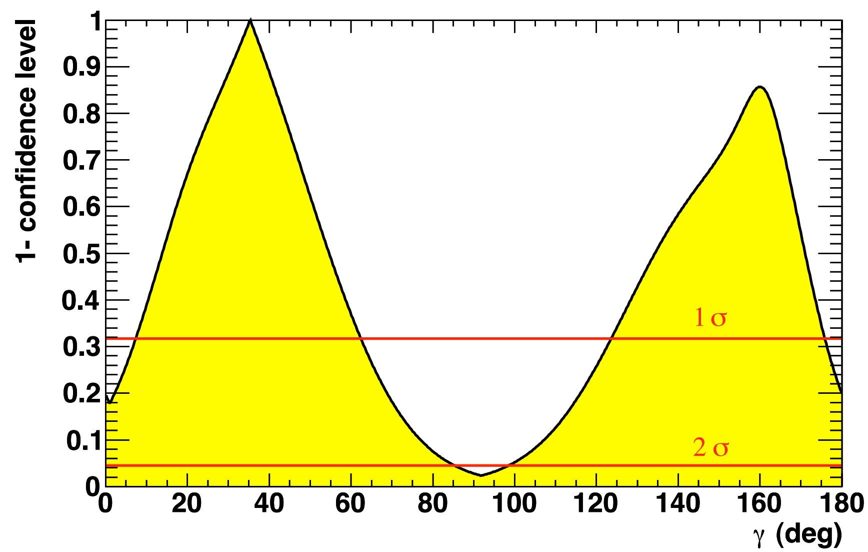
Aubert et al. (BaBar), PRD 80, 092001 (2009) ***[345 fb^{-1}]***

To constrain ϕ_3, combine both GLW and ADS results:

95% CL
1σ (striped)



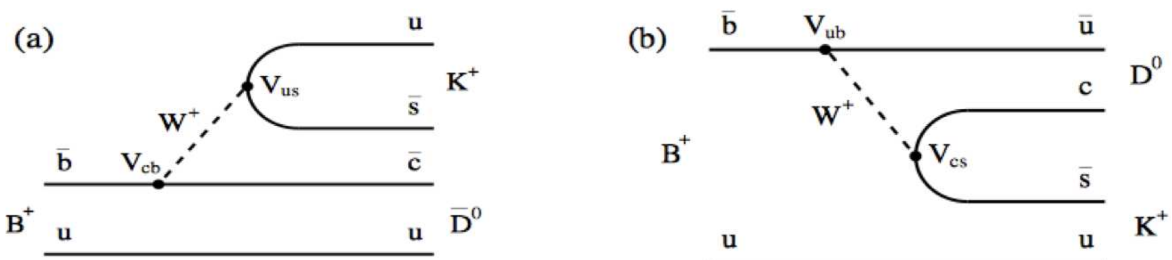




Measurement of ϕ_3 with a Dalitz plot analysis:

Giri, Grossman, Soffer, and Zupan, PRD 68, 054018 (2003);
Bondar, Proc. of BINP Anal. Meeting on Dalitz Analysis, 24-26 Sept. 2002

For the "common" mode (reachable from both D^0 and D^0), use a 3-body final f state such that the formalism for extracting ϕ_3 can be applied to every point in the Dalitz plot; this increases the sensitivity. Best sensitivity is for f = $K_S\pi^+\pi^-$



$$M_+ = A(m_+^2, m_-^2) + re^{i(\delta+\phi_3)} A(m_-^2, m_+^2)$$

$$M_- = A(m_-^2, m_+^2) + re^{i(\delta-\phi_3)} A(m_+^2, m_-^2)$$

$$m_+ = m(K_s^0, \pi^+)$$
$$m_- = m(K_s^0, \pi^-)$$
$$r = \left|\frac{A(B^+ \to D^0 K^+)}{A(B^+ \to \overline{D}^0 K^+)}\right| \sim 0.1-0.2$$

$$|M_\pm|^2 = (r^2)_- |A(m_+^2, m_-^2)|^2 + (r^2)_+ |A(m_-^2, m_+^2)|^2 + 2|A(m_+^2, m_-^2)||A(m_-^2, m_+^2)| \, r\cos(\delta + \theta_{(m_+^2, m_-^2)} \pm \phi_3)$$

Amplitude $A(m_+^2, m_-^2)$ determined from $D^0 \to K_S^0\pi^+\pi^-$ Dalitz plot (continuum production)

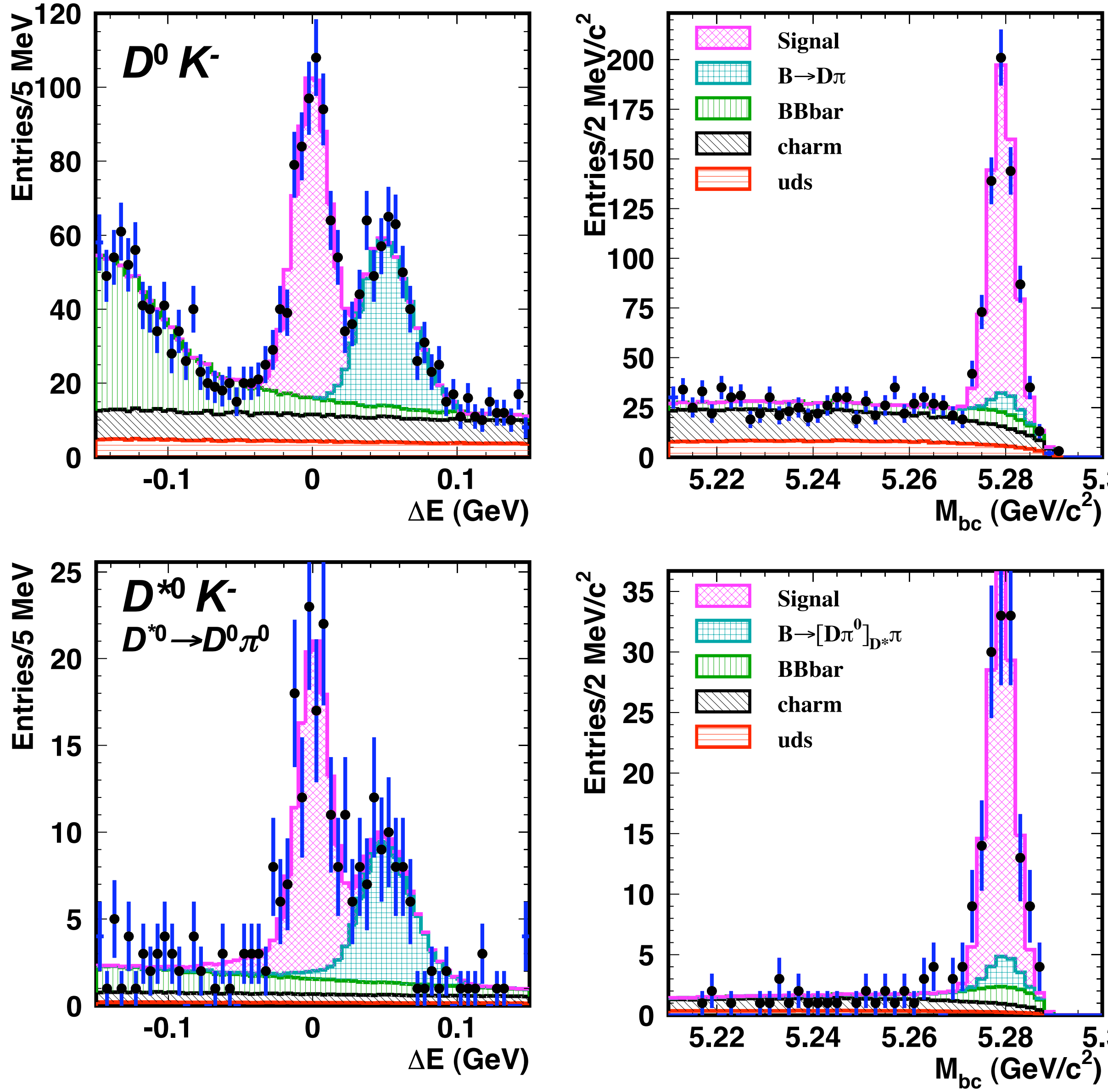
Measurement of ϕ_3 via Dalitz plot analysis:

Poluektov et al. (Belle), PRD 81, 112002 (2010) ***[605 fb^{-1}]***

To increase statistics, use $B^- \to D^0 K^-$ and $B^- \to D^{*0} K^-$, with $D^{*0} \to D^0\pi^0$ and $D^{*0} \to D^0\gamma$ (in all cases $D^0 \to K_S\, \pi^+\pi^-$)

For $B^- \to D^0 K^-$ (most sensitivity), require $M_{bc} > 5.27$ GeV/c^2, $|\Delta E| < 30$ MeV.
Yield: 756 events, 70% purity

The decay model is determined from continuum $D^{*+} \to D^0\pi^+$, $D^0 \to K_S\, \pi^+\pi^-$; this gives a sample of 290k events with purity of 99%.



Measurement of ϕ_3 via Dalitz plot analysis:

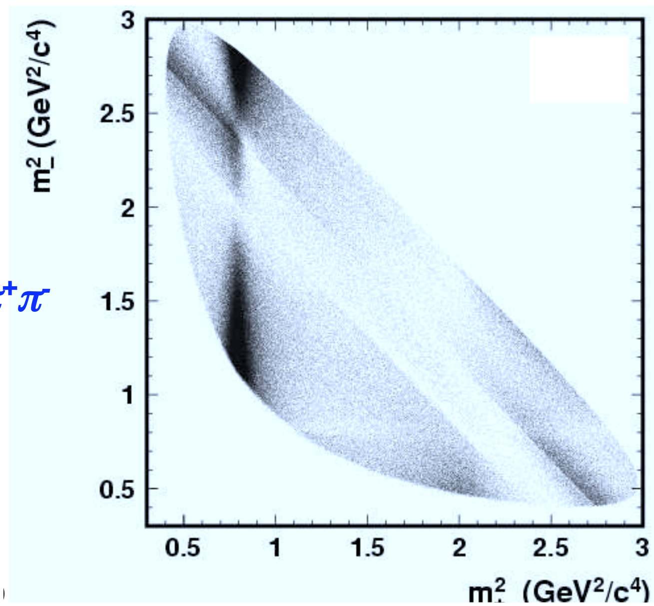
Poluektov et al. (Belle), PRD 81, 112002 (2010); PRD 73, 112009 (2006)

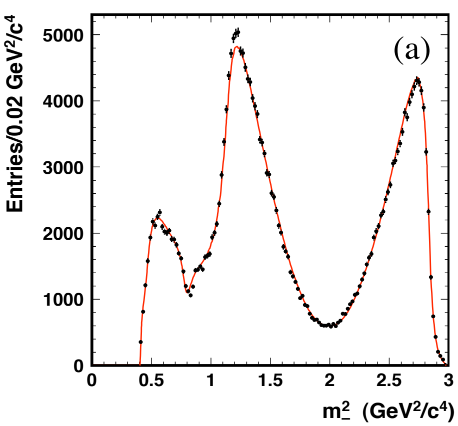
Fit Dalitz plot with Isobar model:

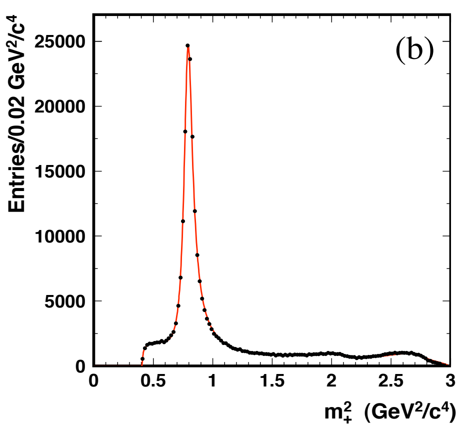
$$f(m_+^2, m_-^2) \;=\; \sum_{j=1}^{N} a_j\, e^{i\varphi_j} \mathcal{A}_j(m_+^2, m_-^2) \;+\; a_{NR} e^{i\varphi_{NR}}$$

$$\text{Fit fraction } f \;\equiv\; \frac{\int\int |a_j\, e^{i\varphi_j} \mathcal{A}_j(m_+^2, m_-^2)|^2\, dm_+^2\, dm_-^2}{\int\int \left|\sum a_j\, e^{i\varphi_j} \mathcal{A}_j(m_+^2, m_-^2)\right|^2 dm_+^2\, dm_-^2}$$

$D^0 \to K_S\, \pi^+\pi^-$ control sample:





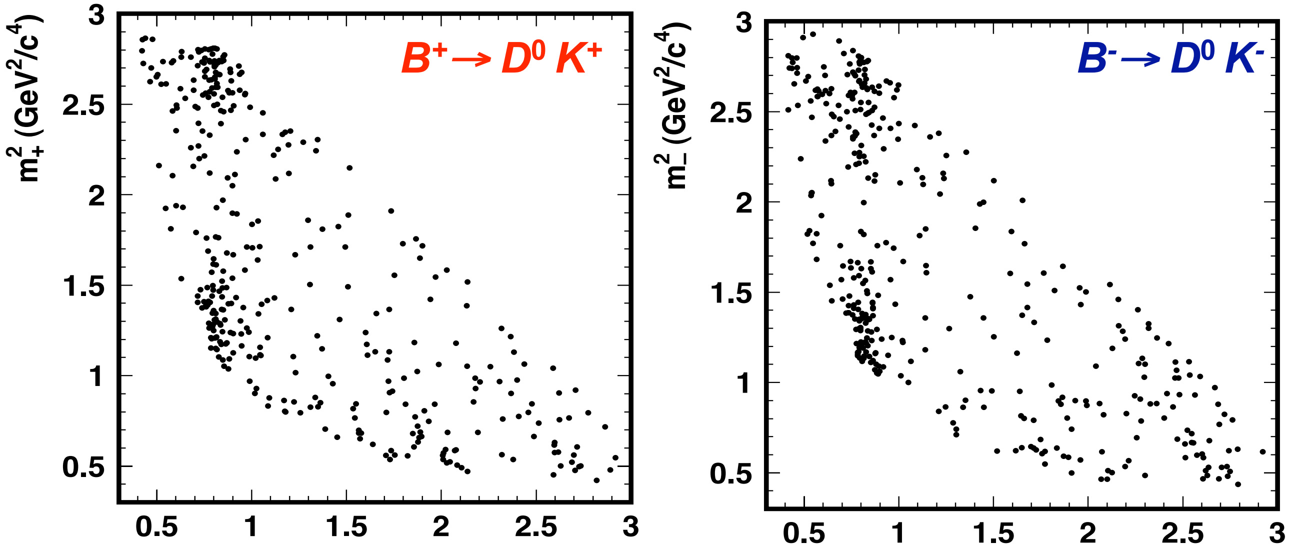


Intermediate state	Amplitude	Phase (°)	Fit fraction (%)
$K_S\sigma_1$	1.56 ± 0.06	214 ± 3	11.0 ± 0.7
$K_S\rho^0$	1.0 (fixed)	0 (fixed)	21.2 ± 0.5
$K_S\omega$	0.0343 ± 0.0008	112.0 ± 1.3	0.526 ± 0.014
$K_S f_0(980)$	0.385 ± 0.006	207.3 ± 2.3	4.72 ± 0.05
$K_S\sigma_2$	0.20 ± 0.02	212 ± 12	0.54 ± 0.10
$K_S f_2(1270)$	1.44 ± 0.04	342.9 ± 1.7	1.82 ± 0.05
$K_S f_0(1370)$	1.56 ± 0.12	110 ± 4	1.9 ± 0.3
$K_S\rho^0(1450)$	0.49 ± 0.08	64 ± 11	0.11 ± 0.04
$K^*(892)^+\pi^-$	1.638 ± 0.010	133.2 ± 0.4	62.9 ± 0.8
$K^*(892)^-\pi^+$	0.149 ± 0.004	325.4 ± 1.3	0.526 ± 0.016
$K^*(1410)^+\pi^-$	0.65 ± 0.05	120 ± 4	0.49 ± 0.07
$K^*(1410)^-\pi^+$	0.42 ± 0.04	253 ± 5	0.21 ± 0.03
$K_0^*(1430)^+\pi^-$	2.21 ± 0.04	358.9 ± 1.1	7.93 ± 0.09
$K_0^*(1430)^-\pi^+$	0.36 ± 0.03	87 ± 4	0.22 ± 0.04
$K_2^*(1430)^+\pi^-$	0.89 ± 0.03	314.8 ± 1.1	1.40 ± 0.06
$K_2^*(1430)^-\pi^+$	0.23 ± 0.02	275 ± 6	0.093 ± 0.014
$K^*(1680)^+\pi^-$	0.88 ± 0.27	82 ± 17	0.06 ± 0.04
$K^*(1680)^-\pi^+$	2.1 ± 0.2	130 ± 6	0.30 ± 0.07
non-resonant	2.7 ± 0.3	160 ± 5	5.0 ± 1.0

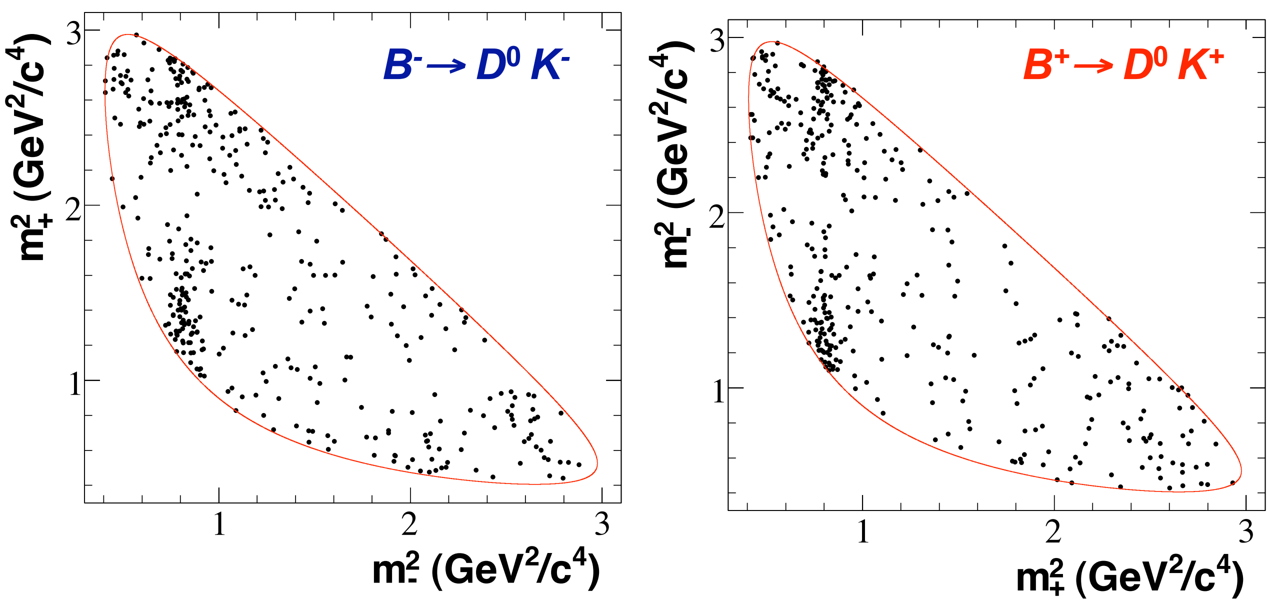
Measurement of ϕ_3 via Dalitz plot analysis:

Signal Dalitz plots ($D^0 \to K_S \pi^+ \pi^-$):

605 fb^{-1}
756 evnts
70% purity



351 fb^{-1}
610 evnts



Fitting:

Unbinned maximum likelihood (ML) fits to separate B^+ and B^- samples. Fitted variables are m_+, m_-, M_{bc}, ΔE, $\cos\theta_{th}$, and a Fisher discr. $\mathcal{F}$.

For $B^- \to D^0 K^-$ four types of backgrounds are considered: continuum u,d,s: continuum c; BB except $D^0\pi^-$; and $D^0\pi^-$. Levels are obtained from M_{bc}-ΔE fit; Dalitz plot shapes from MC.

Measurement of ϕ_3 via Dalitz plot analysis:

Define "Cartesian" fitting parameters:

$B^+ \to D^0 K^+$

$$x_+ = r_+ \cos(\delta + \phi_3)$$
$$y_+ = r_+ \sin(\delta + \phi_3)$$

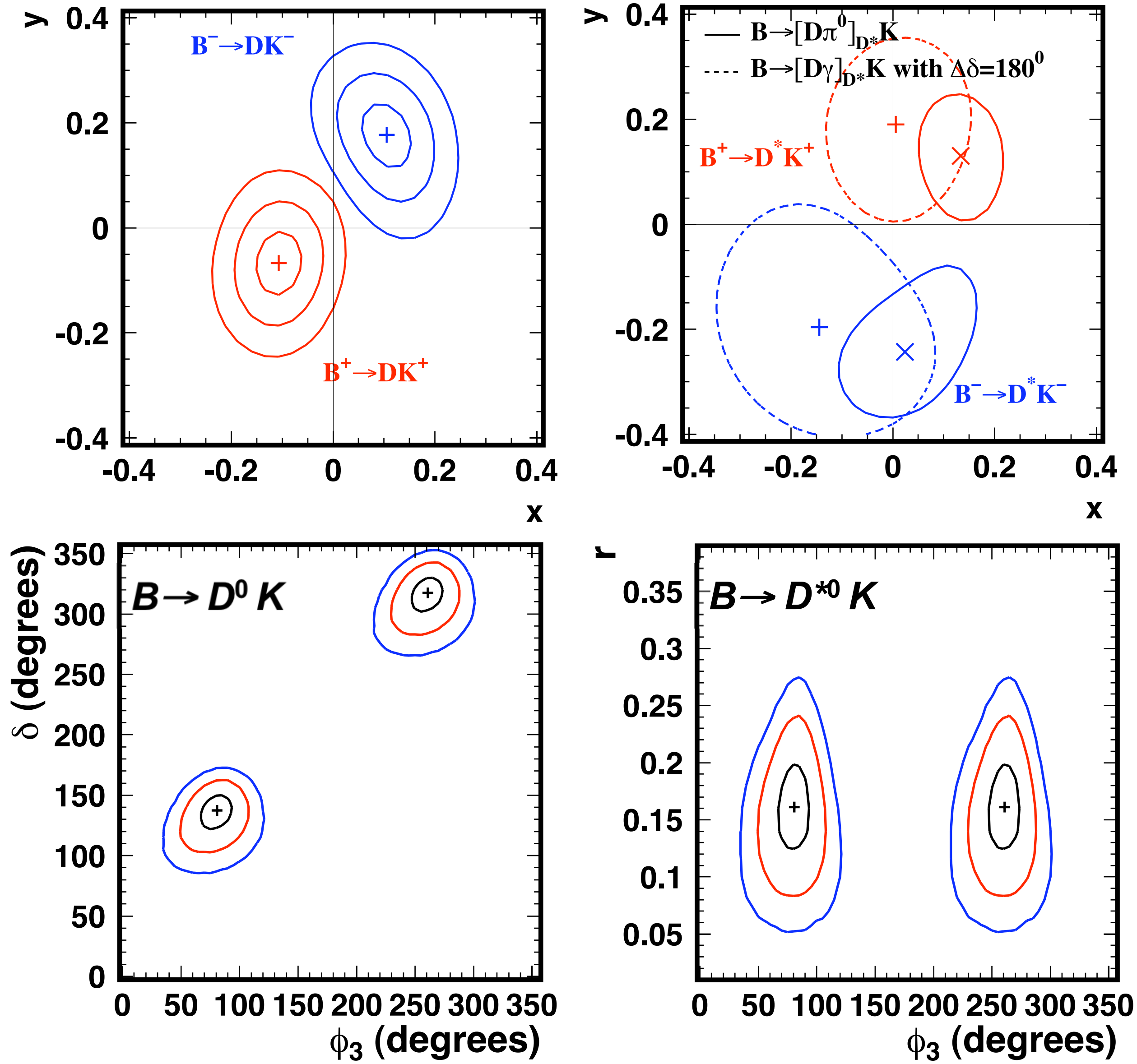
$B^- \to D^0 K^-$

$$x_- = r_- \cos(\delta - \phi_3)$$
$$y_- = r_- \sin(\delta - \phi_3)$$

These fitted parameters have low fitting bias, negligible correlation, and are Gaussian-distributed.

Price: *need additional procedure to translate into confidence intervals for physical parameters r, δ, ϕ_3. Belle constructs frequentist confidence belts and uses Feldman-Cousins procedure.*

Fit results:



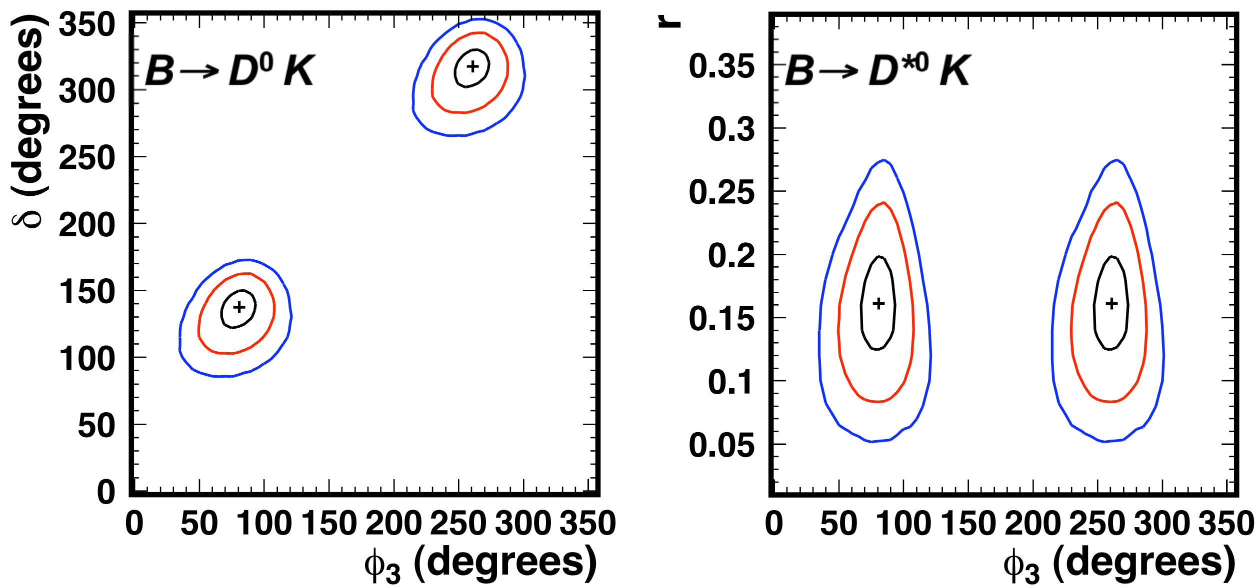
Measurement of ϕ_3 via Dalitz plot analysis:

$B^+ \to D^0 K^+$

$$x_+ = r_+ \cos(\delta + \phi_3)$$
$$y_+ = r_+ \sin(\delta + \phi_3)$$

$B^- \to D^0 K^-$

$$x_- = r_- \cos(\delta - \phi_3)$$
$$y_- = r_- \sin(\delta - \phi_3)$$



Final results for $B \to D^0 K$ ***[605 fb^{-1}] :***

$$x_+ = -0.107 \pm 0.043 \pm 0.011$$
$$y_+ = -0.067 \pm 0.059 \pm 0.018$$

$$x_- = +0.105 \pm 0.047 \pm 0.011$$
$$y_- = +0.177 \pm 0.060 \pm 0.018$$

decay model error

$$\phi_3 = (80.8\,^{+13.1}_{-14.8} \pm 5.0 \pm 8.9)^\circ$$
$$r = 0.161\,^{+0.040}_{-0.038} \pm 0.011\,^{+0.050}_{-0.010}$$
$$\delta = (137.4\,^{+13.0}_{-15.7} \pm 4.0 \pm 22.9)^\circ$$

Results for $B \to D^{*0} K$:

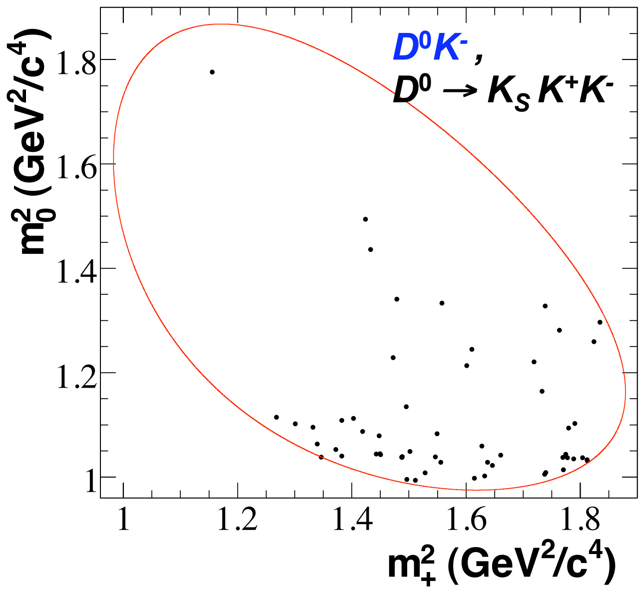
$$\phi_3 = (73.9\,^{+18.9}_{-20.2} \pm 4.2 \pm 8.9)^\circ$$
$$r = 0.196\,^{+0.073}_{-0.072} \pm 0.013\,^{+0.062}_{-0.012}$$
$$\delta = (341.7\,^{+18.6}_{-20.9} \pm 3.2 \pm 22.9)^\circ$$

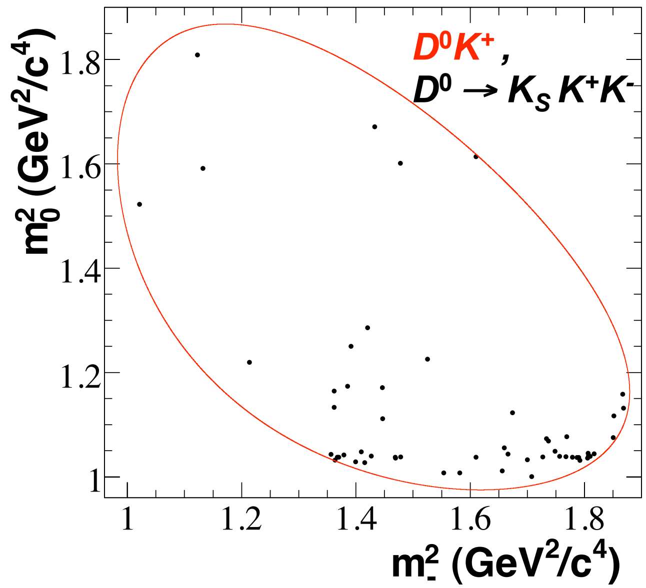
$$\phi_3 = (78.4\,^{+10.8}_{-11.6})^\circ$$

Measurement of ϕ_3 via Dalitz plot analysis:

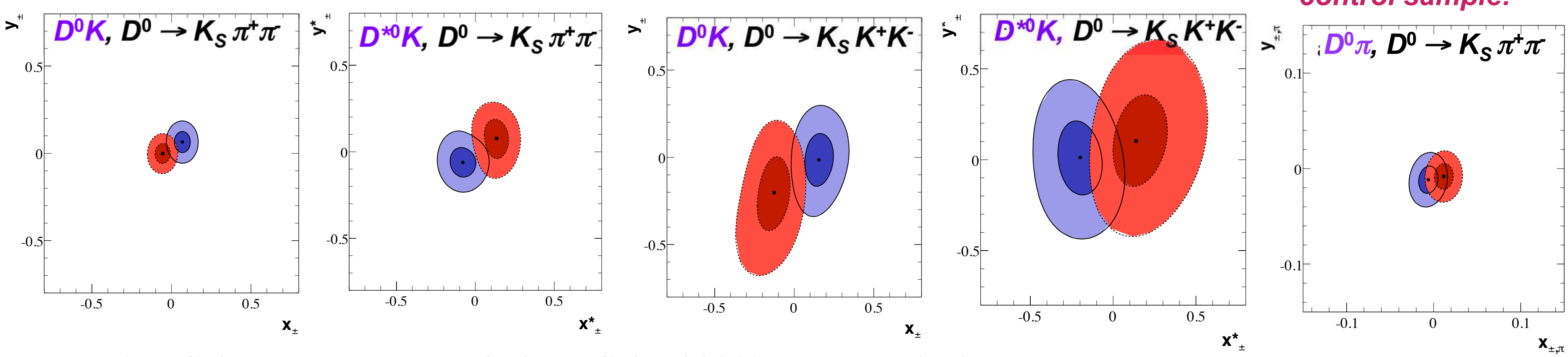
Aubert et al. (BaBar), PRD 78, 034023 (2008) [351 fb^{-1}]

To maximize statistics, use $B^- \to D^0 K^-$ and $B^- \to D^{*0} K^-$ ($D^{*0} \to D^0\pi^0$, $D^0\gamma$) with $D^0 \to K_S \pi^+\pi^-$ and $D^0 \to K_S K^+K^-$. Also $B^- \to D^{(*)0} K^{*-}$ ($D^0 \to K_S \pi^+\pi^-$ only)



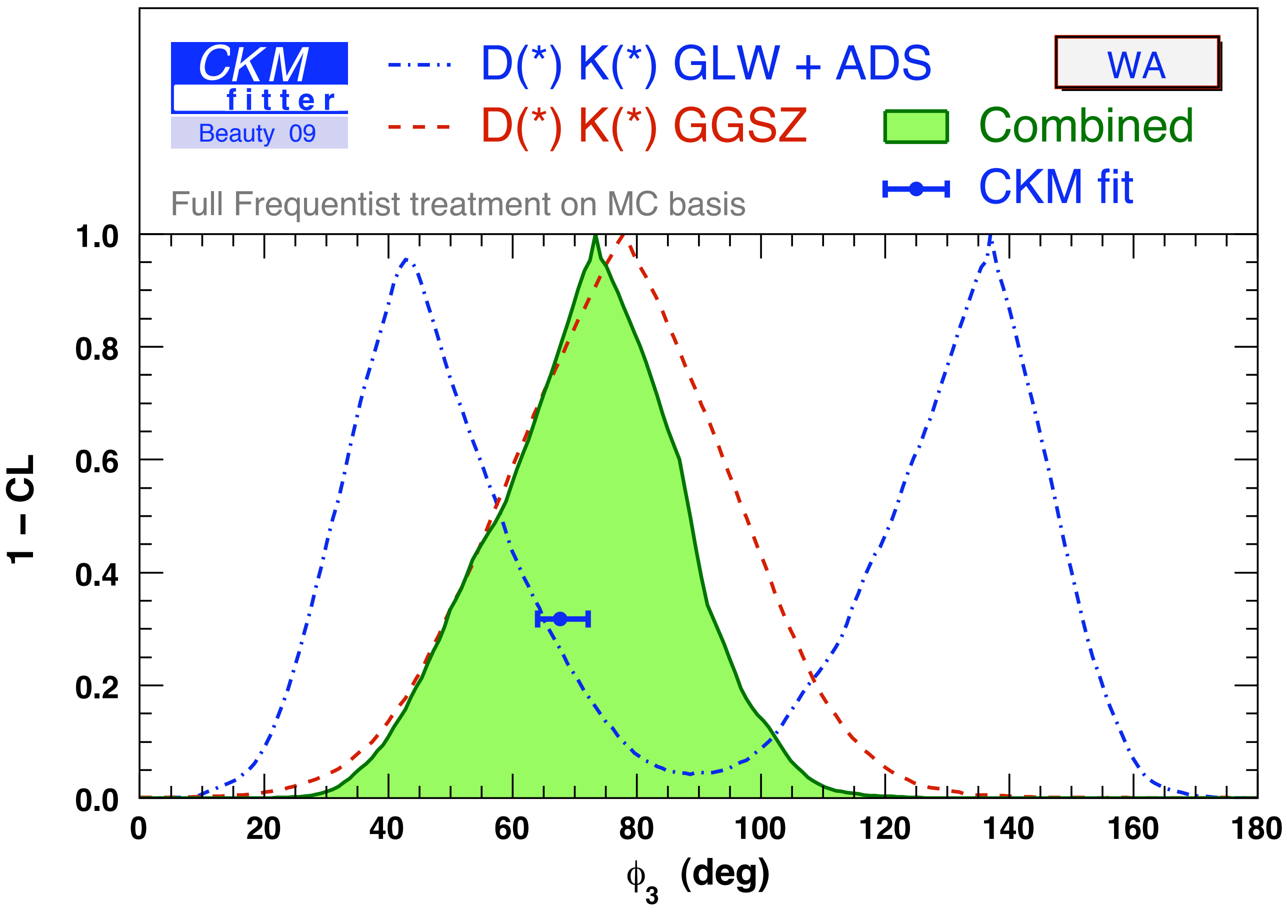


control sample:



Measurement of ϕ_3 via GLW, ADS, Dalitz:

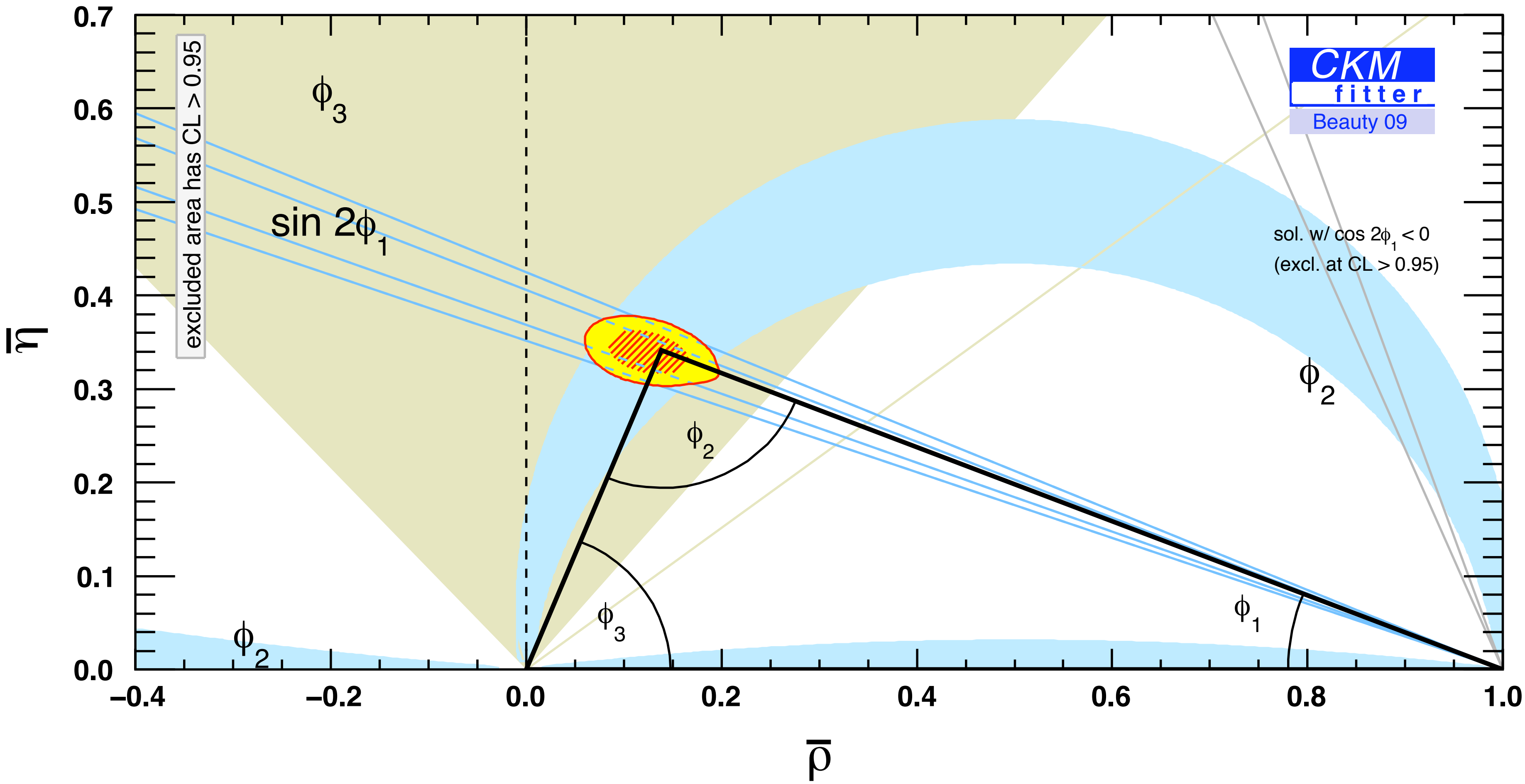
Combining all data (by CKM fitter group, *ckmfitter.in2p3.fr* **):**



$\gamma_{[combined]} = (73^{+22}_{-25})^{\circ}$

The Unitarity Triangle from angles only:

Triangle consistent with closure: $V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$



***BUT:* measurement of triangle sides has some "tension" ⇒ next lecture...**

Measuring decay phases

Another way to test phases, test for new amplitudes:
search for
direct CP violation

$$\begin{aligned} \mathcal{A}_t &= |A_t|\, e^{i\phi_t}\, e^{i\delta_t} \\ \mathcal{A}_p &= |A_p|\, e^{i\phi_p}\, e^{i\delta_p} \end{aligned}$$

$$\begin{aligned} \Gamma(B \to f) &= |\mathcal{A}_t + \mathcal{A}_p|^2 \\ &= |A_t|^2 + |A_p|^2 + 4|A_t||A_p| \cos(\Delta\phi + \Delta\delta), \end{aligned}$$

$$\begin{pmatrix} \Delta\phi & = & \phi_t - \phi_p \\ \Delta\delta & = & \delta_t - \delta_p \end{pmatrix}$$

$$\begin{aligned} \bar{\mathcal{A}}_t &= |A_t|\, e^{-i\phi_t}\, e^{i\delta_t} \\ \bar{\mathcal{A}}_p &= |A_p|\, e^{-i\phi_p}\, e^{i\delta_p} \end{aligned}$$

$$\begin{aligned} \Gamma(\bar{B} \to \bar{f}) &= \left|\bar{\mathcal{A}}_t + \bar{\mathcal{A}}_p\right|^2 \\ &= |A_t|^2 + |A_p|^2 + 4|A_t||A_p| \cos(-\Delta\phi + \Delta\delta), \end{aligned}$$

$$\Rightarrow \quad A_{CP} \equiv \frac{\Gamma(\bar{B} \to \bar{f}) - \Gamma(B \to f)}{\Gamma(\bar{B} \to \bar{f}) + \Gamma(B \to f)} \propto \sin\Delta\phi\, \sin\Delta\delta$$